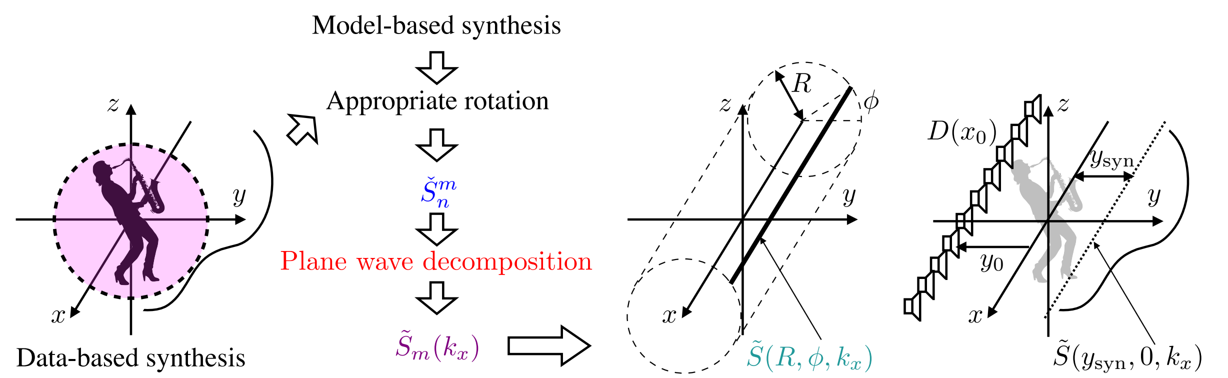
Angular spectrum decomposition-based 2.5D higher-order spherical harmonic sound field synthesis with a linear loudspeaker array

Takuma OKAMOTO, National Institute of Information and Communications Technology, Japan

1. Introduction

- Analytical approaches to sound field recording and reproduction
  - Wave field synthesis (WFS), Spectral division method (SDM): Planer and linear microphone / loudspeaker arrays
  - Higher-order Ambisonics (HOA): Spherical and circular arrays
- Analytical methods for interior sound field recording and synthesis with flexible arrangements of arrays
  - 2.5D WFS for spherical harmonic spectrums: Ahrens *et al*. JASA 2012.
  - 2.5D HOA for angular spectrums: Okamoto ICASSP 2016.
- Proposal: analytical approach for exterior sound field
  - 2.5D SDM for exterior sound field described by spherical harmonic spectrums
    - Recording with a surrounding spherical array, and synthesis with a linear array
    - Applications: listening location informed situations, such as cinema

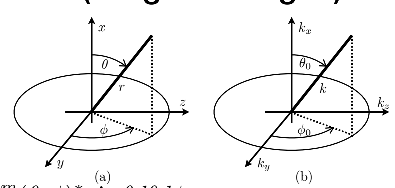


Flowchart of proposed approach

2. Proposed formulation

- Intermediator: Plane wave decomposition of 3D sound field (Herglotz integral)

$$S(r,\theta,\phi) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} \overline{S}(\theta_0,\phi_0)e^{j\boldsymbol{k}^{\mathrm{T}}\boldsymbol{x}}\sin\theta_0 d\theta_0 d\phi_0$$



- Spherical harmonic expansion of 3D sound field

$$S(r,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \check{S}_n^m h_n(kr) Y_n^m(\theta,\phi) \qquad \check{S}_n^m = \frac{1}{h_n(kr)}\int_0^{2\pi}\int_0^{\pi} S(r,\theta,\phi)Y_n^m(\theta,\phi)^* \sin\theta d\theta d\phi$$

$$Y_n^m(\theta,\phi) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}}P_n^{|m|}(\cos\theta)e^{jm\phi} \qquad e^{j\boldsymbol{k}^{\mathrm{T}}\boldsymbol{x}} = 4\pi\sum_{n=0}^{\infty}\sum_{m=-n}^{n} j^n h_n(kr)Y_n^m(\theta,\phi)Y_n^m(\theta_0,\phi_0)^*$$

$$\check{S}_n^m = j^n\int_0^{2\pi}\int_0^{\pi}\overline{S}(\theta_0,\phi_0)Y_n^m(\theta_0,\phi_0)^*\sin\theta_0 d\theta_0 d\phi_0 \qquad \overline{S}(\theta_0,\phi_0) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} j^{-n}\check{S}_n^m Y_n^m(\theta_0,\phi_0) = \sum_{m=-\infty}^{\infty}\sum_{n=|m|}^{\infty} j^{-n}\check{S}_n^m Y_n^m(\theta_0,0)e^{jm\phi_0}$$

- 3D cylindrical harmonic expansion of 3D sound field

$$S(R,\phi,x) = \sum_{m=-\infty}^{\infty} e^{jm\phi}\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{S}_m(k_x)H_m(k_rR)e^{jk_xx}dk_x \qquad e^{j\boldsymbol{k}^{\mathrm{T}}\boldsymbol{x}} = \sum_{m=-\infty}^{\infty} e^{jm(\phi-\phi_0)}\frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi j^m H_m(k_rR)\delta(k_x - k\cos\theta_0)e^{jk_xx}dk_x$$

$$\tilde{S}_m(k_x) = \frac{j^m}{2}\int_0^{2\pi}\int_0^{\pi}\overline{S}(\theta_0,\phi_0)e^{-jm\phi_0}\delta(k_x-k\cos\theta_0)\sin\theta_0 d\theta_0 d\phi_0 \qquad k_x = k\cos\theta_0, \quad \sqrt{k^2-k_x^2} = k_r = k\sin\theta_0$$

- Analytical conversion from spherical to 3D cylindrical harmonic spectrums

$$\tilde{S}_m(k_x) = \frac{j^m}{2}\int_0^{2\pi}\int_k^{-k}\overline{S}(\theta_0,\phi_0)e^{-jm\phi_0}\delta(k_x-k\cos\theta_0)\frac{k_r}{k}\frac{d}{dk_x}\left\{\cos^{-1}\left(\frac{k_x}{k}\right)\right\}dk_x d\phi_0$$

with $d\theta_0 \to dk_x$

$$= \frac{j^m\pi}{k}\sum_{n=|m|}^{\infty} j^{-n}\check{S}_n^m Y_n^m(k_x) \qquad \frac{d}{dk_x}\left\{\cos^{-1}\left(\frac{k_x}{k}\right)\right\} = \frac{-1}{\sqrt{k^2-k_x^2}} = -\frac{1}{k_r}$$

$$Y_n^m(k_x) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}}P_n^{|m|}\left(\frac{k_x}{k}\right) \qquad \frac{1}{2\pi}\int_0^{2\pi}\overline{S}(\theta_0,\phi_0)e^{-jm\phi_0}d\phi_0 = \sum_{n=|m|}^{\infty} j^{-n}\check{S}_n^m Y_n^m(k_x)$$

- Analytical conversion into angular spectrums

$$\tilde{S}(R,\phi,k_x) = \sum_{m=-\infty}^{\infty}\tilde{S}_m(k_x)H_m(k_rR)e^{jm\phi} = \frac{\pi}{k}\sum_{m=-\infty}^{\infty} j^m H_m(k_rR)e^{jm\phi}\sum_{n=|m|}^{\infty} j^{-n}\check{S}_n^m Y_n^m(k_x)$$

3. Analytical driving function for 2.5D SDM

- Sound pressure synthesized by a linear sound source

$$S(\boldsymbol{x}) = \int_{-\infty}^{\infty} D(\boldsymbol{x}_0)G_{3\mathrm{D}}(\boldsymbol{x},\boldsymbol{x}_0)dx_0 \xrightarrow{\mathcal{F}} \tilde{S}(y_{\mathrm{syn}},0,k_x) = \tilde{D}(k_x)\tilde{G}_{3\mathrm{D}}(k_x,y,y_0) = \tilde{D}(k_x)\frac{j}{4}H_0\left(k_r(y_{\mathrm{syn}}-y_0)\right)$$

- Analytical driving function for 2.5D SDM

$$\tilde{D}(k_x) = \frac{-4j\pi}{kH_0\left(k_r(y_{\mathrm{syn}}-y_0)\right)}\cdot\sum_{m=-\infty}^{\infty} j^m H_m(k_r y_{\mathrm{syn}})\sum_{n=|m|}^{\infty} j^{-n}\check{S}_n^m Y_n^m(k_x)$$

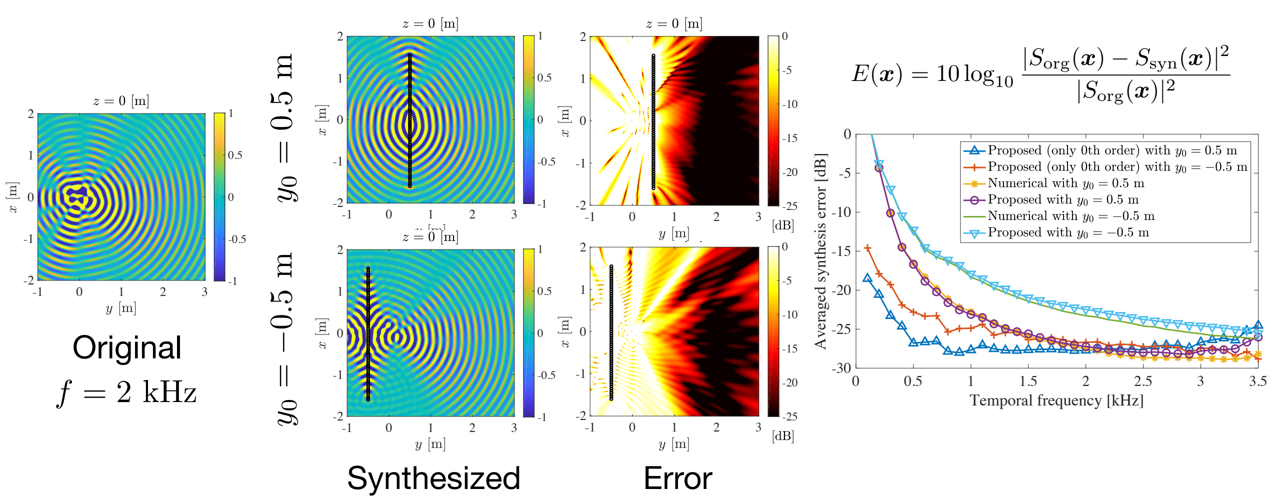
- Comparison with focused point source synthesis by 2.5D SDM

$$\tilde{D}(k_x)_{\mathrm{ps}} = \tilde{P}_{\mathrm{ps}}e^{-jk_xx_{\mathrm{s}}}\frac{H_0\left(k_r(y_{\mathrm{syn}}-y_{\mathrm{s}})\right)}{H_0\left(k_r(y_{\mathrm{syn}}-y_0)\right)}$$

Equivalent to proposal when $\tilde{P}_{\mathrm{ps}} = -\frac{j\sqrt{4\pi}\check{S}_0^0}{k}$, $m = 0$, $x_{\mathrm{s}} = y_{\mathrm{s}} = 0$

4. Computer simulations

- Simulation condition
  - Original sound field: modeled by 81 random numbers up to $n = 8$
  - 64 loudspeakers with $\Delta x = 0.05$ m and $y_{\mathrm{syn}} = 2.0$ m



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