Analytical methods of generating multiple sound zones for open and baffled circular loudspeaker arrays

Takuma OKAMOTO, National Institute of Information and Communication Technology, Japan

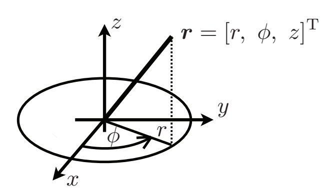
1. Introduction

- **Personalizing listening areas using multiple loudspeakers**
  - Multiple sound zones with a linear array (*e.g.* T. Okamoto@ICASSP 2014.)
    - Property: spectral division method based analytical solution
    - Problem: many loudspeakers and large production space are required
  - Motivation: Deriving analytical solutions for compact arrays
- **Conventional methods with circular arrays and their problems**
  - Least squares (LS) methods (*e.g.* T. Betlehem *et al.* 2006.)
    - Very ill-conditioned and unstable driving signals of loudspeakers
  - Beamforming methods using a baffled circular array (*e.g.* F. M. Fazi *et al.* 2014.)
    - Far field approximation is introduced and near field cannot be controlled
- **Proposed methods for open and baffled circular loudspeaker arrays**
  - Analytical solutions for circular monopole sound source distributions
    - Precise solutions are derived based on 2.5D sound field representation
    - Bright and dark zones are directly modeled by a rectangular window

2. 2.5D sound field produced by circular sources

- **3D sound field produced by a cylindrical sound source**

$$P(\boldsymbol{r},\omega)=\int_0^{2\pi}\int_{-\infty}^{\infty}D(\boldsymbol{r}_0,\omega)G(\boldsymbol{r},\boldsymbol{r}_0,\omega)r_0dz_0d\phi_0$$



  - 3D sound field propagated by an open cylindrical sound source described by 3D cylindrical harmonics expansion

$$G_{\text{open}}(\boldsymbol{r},\boldsymbol{r}_0,\omega)=\frac{e^{jk|\boldsymbol{r}-\boldsymbol{r}_0|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}_0|}=\sum_{n=-\infty}^{\infty}\underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{j}{4}J_n(k_r r_0)H_n^{(1)}(k_r r)dk_z}_{\mathring{G}_{\text{open},n}(r>r_0,\omega)}e^{jk_z(z-z_0)}e^{jn(\phi-\phi_0)}\quad \text{for } r>r_0$$

  - 3D sound field propagated by a baffled cylindrical sound source described by 3D cylindrical harmonics expansion

$$G_{\text{baffled}}(\boldsymbol{r},\boldsymbol{r}_0,\omega)=\sum_{n=-\infty}^{\infty}\underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{-H_n^{(1)}(k_r r)}{2\pi k_r r_0 H_n^{(1)\prime}(k_r r_0)}dk_z}_{\mathring{G}_{\text{baffled},n}(r>r_0,\omega)}e^{jk_z(z-z_0)}e^{jn(\phi-\phi_0)}\quad \text{for } r>r_0$$

- **Driving function of circular monopole sound source distributions described by 2.5D sound field representation**

$$P(\boldsymbol{r},\omega)=\int_0^{2\pi}D(r_0,\phi_0,z=0,\omega)G(\boldsymbol{r},\boldsymbol{r}_0,\omega)r_0d\phi_0 \quad \xrightarrow{\mathcal{F}} \quad \mathring{D}_n(r_0,\omega)=\frac{\mathring{P}_n(r,\omega)}{2\pi r_0\mathring{G}_n(r,r_0,\omega)}$$

3. Proposed method

- **Analytical solution of modeling sound pressure by a rectangular window**
  - Spatial Fourier series expansion of sound pressure

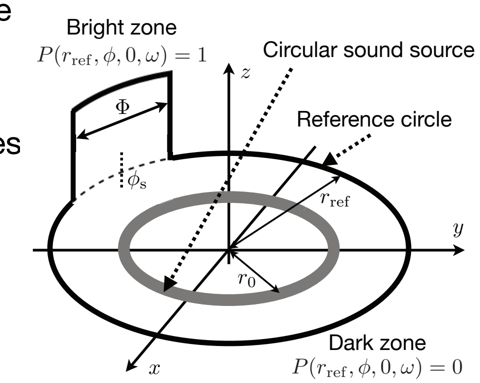
$$\mathring{P}_n(\Phi,\phi_{\text{S}},\omega)=\Phi\text{sinc}\left(\frac{n\Phi}{2\pi}\right)e^{-jn\phi_{\text{s}}}$$

  - Spatial filtering for generating multiple sound zones

$$\mathring{F}_n(r_0,r_{\text{ref}},\Phi,\phi_{\text{s}},\omega)=\frac{\Phi\text{sinc}(n\Phi/2\pi)e^{-jn\phi_{\text{s}}}}{2\pi r_0\mathring{G}_n(r_{\text{ref}}>r_0,\omega)}$$

  - Filter coefficients in temporal frequency domain

$$F(r_0,\phi_l,\omega)=\sum_{n=-N}^{N}\mathring{F}_n(r_0,\omega)e^{jn\phi_l}$$

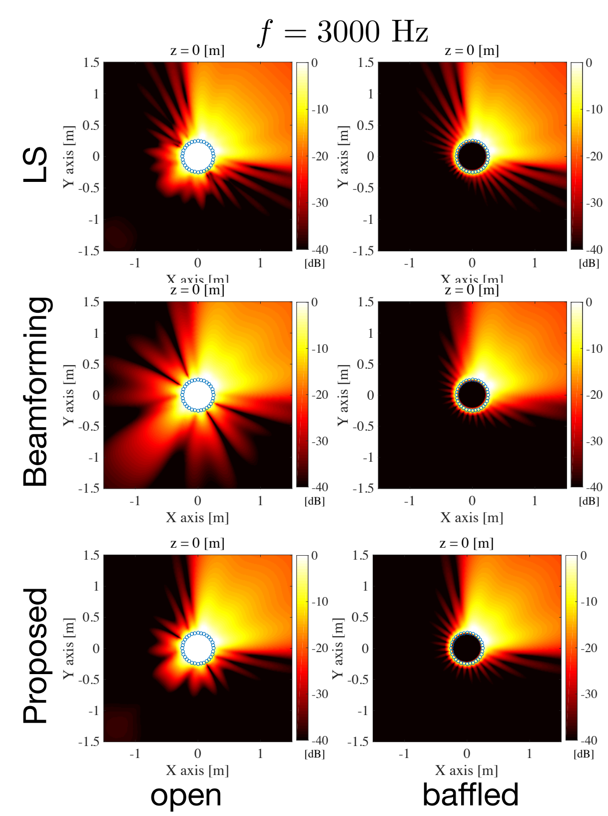


  - Discarding evanescent wave component for calculating stable driving signals

$$N=\begin{cases}\lfloor(L-1)/2\rfloor, & \text{for}\quad \lfloor|kr_0|\rfloor\geq\lfloor(L-1)/2\rfloor\\ \lfloor|kr_0|\rfloor, & \text{for}\quad \lfloor|kr_0|\rfloor<\lfloor(L-1)/2\rfloor\end{cases}$$

4. Computer simulations

- **Produced sound pressure level and bright to dark ratio**

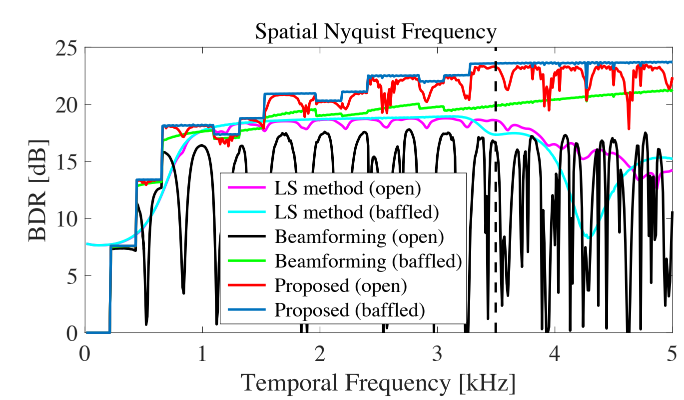


$$F_{\text{BF,open}}=\sum_{n=-N}^{N}\frac{\Phi\text{sinc}(n\Phi/2\pi)e^{jn(\phi_l-\phi_{\text{S}})}}{2\pi j^{-n}J_n(kr_0)}$$

$$F_{\text{BF,baffled}}=\sum_{n=-N}^{N}\frac{kr_0H_n'(kr_0)\Phi\text{sinc}(n\Phi/2\pi)e^{jn(\phi_l-\phi_{\text{s}})}}{4j^{(1-n)}}$$

$$BDR(\omega)=20\log_{10}\frac{\sum_{\boldsymbol{r}_{\text{b}}}|P(\boldsymbol{r}_{\text{b}},\omega)|/\Phi_{\text{b}}}{\sum_{\boldsymbol{r}_{\text{d}}}|P(\boldsymbol{r}_{\text{d}},\omega)|/\Phi_{\text{d}}}$$

with $\Phi=\pi/2$, $r_0=0.25$ m and $r_{\text{ref}}=1.0$ m



This study was partly supported by JSPS KAKENHI Grant Number 15K21674.