CLOSE-TALKING RECORDING WITH PLANARLY DISTRIBUTED MICROPHONES

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ABSTRACT

This paper provides a close-talking recording method with microphones in arbitrary planar distributions based on sound pressure interpolation. In this method, sound pressures recorded by microphones located near the center of multiple co-centered circular microphone arrays are interpolated using sound pressures recorded by these arrays via multiple 2D cylindrical harmonic analyses. If the sound sources are outside the spherical boundary formed by the maximum radius of the arrays, the interpolation is completed. However, if the sound sources are inside the spherical boundary, the interpolation is not completed in principle. Using this principle, close-talking recording is realized as a residual response between recorded and interpolated sound pressures. The recording radial sensitivity can be controlled by varying the maximum array radius. By introducing a kernel ridge regression-based sound field interpolation method, the approach can be extended to the microphones in arbitrary planar distributions. Simulations confirm the effectiveness of these close-talking recording methods.

***Index Terms*—** Close-talking microphone, sound pressure interpolation, circular microphone array, planar microphone distribution, kernel ridge regression

1. INTRODUCTION

Microphone arrays are typically used for beamforming to record only sound signals that propagate from desired directions and many approaches to this type of recording have been widely investigated [1–5]. For recording of only sound sources located near microphones, however, close-talking recording methods with microphone arrays have also been developed. When compared with close-talking recording methods that are dependent on the source directions [6–13], source localization-free approaches are useful because source localization and tracking are not required. Two source localization-free close-talking methods have been proposed to date.

First, a method that used an open spherical double-layer microphone array combined with a microphone located at the center of the array was proposed [14]. In this method, the sound pressure at the array's center is interpolated from the sound pressures recorded by the array based on the Kirchhoff–Helmholtz integral [15]. If the sound sources are located far from the array, the residual between the sound pressures recorded by the microphone at the array center and interpolated by the array is small. In contrast, if the sound sources are located close to the array, the residual is large. Using this principle, close-talking recording can be realized.

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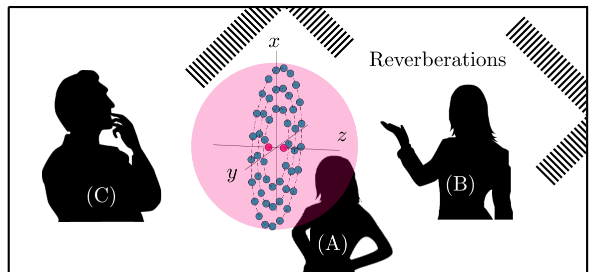


Fig. 1. Conceptual diagram of proposed close-talking recording with microphones in planar distribution rotated to vertical angle for practical implementation.

To reduce the number of microphones required, this approach is realized using an open spherical single-layer microphone array rather than a double-layer array based on sound field interpolation in the spherical harmonic spectrum domain [16]. This method interpolates the sound pressure at the array center from the sound pressures recorded by the array based on the 0-th order spherical harmonic expansion and can realize close-talking recording as per the method using the double-layer array [14] while also reducing the number of microphones by half. This method has been applied to near- and far-field sound source separation [17]. In this approach, a spherical array with a small radius or a dual-sphere microphone array should be introduced [16, 18] to avoid the forbidden frequency problem where the spherical Bessel function becomes zero [15]. However, the principle of close-talking used in these methods is based on incomplete interpolation of the sound field through use of a small number of microphones. Therefore, it is difficult to control the radial sensitivity of the recording. It is also difficult to reduce the number of microphones from a near-equidense distribution on a sphere that is dependent on the target frequency.

To solve this problem, this paper proposes an alternative residual-based close-talking approach using microphones in a planar distribution. In the proposed method, the sound pressure at the center of the array is also interpolated using sound pressures recorded by multiple co-centered circular microphone arrays via multiple 2D cylindrical harmonic analyses [19–21]. If the sound sources are located outside the spherical boundary formed by the maximum radius of the arrays, the interpolation is completed. In contrast, if the sound sources are located inside the spherical boundary, the interpolation is not completed in principle. Using this principle, close-talking recording can be realized as a residual response between the recorded and interpolated sound pressures. When compared with previous methods using spherical arrays [14, 16], the recording radial sensitivity is defined by the maximum radius of the arrays and can be controlled by varying the maximum radius. Additionally, by introducing a kernel ridge regression [22]-based sound field interpolation method [23], the proposed approach can be extended to the method using microphones in arbitrary planar distributions. For actual implementation, the microphones in the planar distribution are rotated to vertical angles, as shown in Fig. 1, where close-talking recording can be realized to only record the voice of speaker (A) because the other sources (speakers (B) and (C), reverberations) are located outside the boundary and the sound pressures from these sources can be completely interpolated.

The proposed method was inspired by localized sound zone generation methods using an open spherical loudspeaker array [24] or multiple co-centered circular loudspeaker arrays [25]. In [24], the sound field radiated from a point source located at the center of the array is cancelled using the open spherical array and a localized sound zone can then be generated inside the array. Similarly, a 3D sound field propagating from a point source located at the center of the array can be cancelled using multiple co-centered arrays [26, 27] and a 3D localized sound zone was also generated inside a sphere with the maximum radius of the arrays in [25]. Interestingly, the filter coefficients of the array used for the method in [24] are equivalent to those used for the close-talking method with an open spherical microphone array [16] based on acoustic source-receiver reciprocity [28]. Application of this reciprocity to the localized sound zone generation method with multiple co-centered circular loudspeaker arrays [25] helped to inspire the proposed close-talking method with multiple co-centered microphone arrays.

To ensure the reproducibility of this study, the MATLAB code used in the computer simulations conducted in Sec. 4 is available online. [1]

2. HORIZONTAL 3D SOUND FIELD RECORDING

2.1. 3D sound field in horizontal plane

The interior expansion of a 3D sound field in a homogeneous region that is free from sources is given by:

$$S(r,\theta,\phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \breve{A}_n^m j_n(kr) Y_n^m(\theta,\phi), \tag{1}$$

where $\breve{A}_n^m$ and j_n are the spherical harmonic spectra of the interior sound field and the n-th order spherical Bessel function, respectively, and k is the wavenumber [15];

$$Y_n^m(\theta,\phi) = \underbrace{\sqrt{\frac{(2n+1)}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}} P_n^{|m|}(\cos\theta)}_{\mathcal{P}_n^{|m|}(\cos\theta)} e^{jm\phi} \tag{2}$$

represents the spherical harmonics, and $P_n^{|m|}$ is the associated Legendre function [29].

To consider a 3D sound field in the horizontal plane with $\theta = \pi/2$, (1) is represented by:

$$S(r,\pi/2,\phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \breve{A}_n^m j_n(kr) \mathcal{P}_n^{|m|}(0) e^{jm\phi}, \tag{3}$$

where only $\breve{A}_n^m$ for $n+|m|$ even is present because $\mathcal{P}_n^{|m|}(0) = 0$ for $n+|m|$ odd [19]. To obtain $\breve{A}_n^m$ for the $n+|m|$ odd components, the vertical derivatives of $\mathcal{P}_n^{|m|}(0)$ are required and can be measured using differential microphones or vertical microphone pairs in the horizontal plane [19]. Therefore, both omni-directional and vertical differential microphones are required to estimate the entire 3D sound field [19]. However, the proposed close-talking approach can set the positions of the interpolated sound pressures freely and these positions can also be located on the horizontal plane. Then, the interpolated sound pressures also contain only $\breve{A}_n^m$ for $n+|m|$ even components and the proposed method does not require the vertical microphone pairs as used in horizontal 3D sound field recording with a planar microphone array [30] for 2.5D higher-order Ambisonics (HOA) [31, 32].

[1] https://codeocean.com/capsule/6915360

2.2. Horizontal 3D sound field recording with multiple co-centered omni-directional circular microphone arrays

Using (3), a continuous circular sound pressure distribution centered at the origin of the x-y plane ($\theta = \pi/2$) can be converted into 2D cylindrical harmonic spectra [15] and represented by:

$$\mathring{S}_m(r) = \frac{1}{2\pi}\int_0^{2\pi} S(r,\pi/2,\phi) e^{-jm\phi} d\phi \tag{4}$$

$$= \sum_{n=|m|}^{\infty} \breve{A}_n^m j_n(kr) \mathcal{P}_n^{|m|}(0). \tag{5}$$

When the maximum spherical harmonic order is N and the radius of the circle is R_q, (5) can be given as

$$\mathring{S}_m(R_q) \simeq \sum_{n=|m|}^{N} \breve{A}_n^m j_n(kR_q) \mathcal{P}_n^{|m|}(0). \tag{6}$$

Introduction of multiple radii ($q = 1, 2, \cdots, Q$) to (6) allows it to be represented in matrix form as:

$$\mathring{\boldsymbol{S}}_m = \boldsymbol{U}_{|m|} \breve{\boldsymbol{A}}_m^{\text{even}}, \tag{7}$$

where

$$\mathring{\boldsymbol{S}}_m = \left[\mathring{S}_m(R_1),\ \mathring{S}_m(R_2),\ \cdots,\ \mathring{S}_m(R_Q)\right]^{\mathsf{T}}, \tag{8}$$

$$\boldsymbol{U}_{|m|} = \begin{bmatrix} U_{|m|}^{|m|}(kR_1) & U_{|m|+2}^{|m|}(kR_1) & \cdots & U_N^{|m|}(kR_1) \\ U_{|m|}^{|m|}(kR_2) & U_{|m|+2}^{|m|}(kR_2) & \cdots & U_N^{|m|}(kR_2) \\ \vdots & \vdots & \vdots & \vdots \\ U_{|m|}^{|m|}(kR_Q) & U_{|m|+2}^{|m|}(kR_Q) & \cdots & U_N^{|m|}(kR_Q) \end{bmatrix}, \tag{9}$$

$$U_n^{|m|}(kR) = j_n(kR)\mathcal{P}_n^{|m|}(0), \tag{10}$$

and

$$\breve{\boldsymbol{A}}_m^{\text{even}} = \left[\breve{A}_{|m|}^m,\ \breve{A}_{|m|+2}^m,\ \cdots,\ \breve{A}_N^m\right]^{\mathsf{T}}, \tag{11}$$

in the case where both N and m are either odd or even; otherwise, replace N in both (9) and (11) with $N-1$. The 3D spherical harmonic spectra $\breve{A}_n^m$ for $n+|m|$ even can then be obtained from multiple co-centered omni-directional circular sound pressure distributions as [19]:

$$\breve{\boldsymbol{A}}_m^{\text{even}} = \boldsymbol{U}_{|m|}^{+} \mathring{\boldsymbol{S}}_m, \tag{12}$$

where $\boldsymbol{U}_{|m|}^{+}$ is the generalized inverse of $\boldsymbol{U}_{|m|}$ [21].

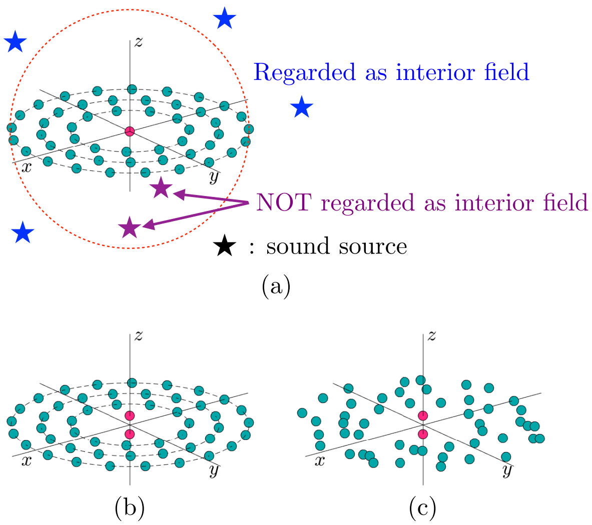


Fig. 2. Proposed close-talking recording method with (a) multiple circular arrays and one additional microphone; (b) multiple circular arrays and a vertical monopole pair; (c) microphones in planar distribution and a vertical monopole pair.

3. PROPOSED CLOSE-TALKING RECORDING METHOD

3.1. Close-talking recording using multiple co-centered circular arrays

In the proposed method, an additional microphone is also set at the center of the multiple circular arrays, as used in the close-talking methods with an open spherical array [14, 16]; this is shown in Fig. 2(a). In this case, all microphones are located on the horizontal plane. The sound pressure recorded using the microphone at the center of the array can then be interpolated from the sound pressures recorded by the multiple circular arrays using (12) and (3). Specifically, by locating the additional microphone at the origin, (3) can be simplified with only the 0-th order component and the residual $D_{0,\mathrm{circ}}$ between the recorded sound pressure and the interpolated pressure is also calculated as [16]:

$$D_{0,\mathrm{circ}} = S(0, \pi/2, 0) - \frac{\breve{A}_0^0}{\sqrt{4\pi}}, \tag{13}$$

where $j_0(0) = 1$ and $\mathcal{P}_0^0(0) = 1/\sqrt{4\pi}$ [15].

As described in Section 2.1, (1) and (3) only hold for an interior sound field in which all sound sources are located outside the spherical recording boundary [15]. In this case, the boundary is a sphere with the maximum radius of the multiple circular arrays, as illustrated in Fig. 2(a). Therefore, if the sound sources are located outside the spherical boundary, $D_{0,\mathrm{circ}}$ is small. In contrast, if the sound sources are inside the boundary, (1) and (3) no longer hold and $D_{0,\mathrm{circ}}$ is large. In the proposed approach, close-talking recording can be realized using this principle. By varying the maximum radius of the array, the recording radial sensitivity can be controlled in principle.

However, when the sound sources are inside but close to the boundary, $D_{0,\mathrm{circ}}$ becomes small because all the microphones are located on the same horizontal plane and $S(0, 0, 0)$ can be predicted using $\breve{A}_0^0/\sqrt{4\pi}$ to some extent, even though the sound sources are inside the boundary. To expand the recording radial sensitivity inside the boundary while maintaining the simplicity of the approach, the proposed method is extended to interpolate the sound pressures recorded using a vertical microphone pair located on the z-axis with $r = d$, $\theta = 0$ and π, as shown in Fig. 2(b). Using this arrangement, the recording remains symmetrical with respect to the horizontal plane and the z-axis. The sum of the recorded sound pressures is thus also symmetrical with respect to the horizontal plane and can be interpolated using the sound pressures recorded by the array, although the vertical pair is not located on the horizontal plane. Additionally, it is sufficient to calculate only the spherical harmonic spectra $\breve{A}_n^0$ because the recording is symmetrical with respect to the z-axis. Therefore, the residual can be derived from (3) and given as

$$D_{\pm d,\mathrm{circ}} = S(d, 0, 0) + S(d, \pi, 0) - \sum_{n=0}^{N} \sqrt{\frac{2n+1}{\pi}} \breve{A}_n^0 j_n(kd), \tag{14}$$

where

$$\mathcal{P}_n^0(1) + \mathcal{P}_n^0(-1) = \sqrt{\frac{2n+1}{\pi}}. \tag{15}$$

3.2. Close-talking recording with microphones in arbitrary planar distributions

The proposed close-talking recording approach can be extended to use arbitrary distributions of microphones in the horizontal plane rather than multiple co-centered circular arrays. Although the estimated spherical harmonic spectra $\breve{A}_n^m$ for $n + |m|$ even are required for 2.5D HOA in [30], the proposed close-talking method ultimately requires the sound pressures $S(0, 0, 0)$ or $S(d, 0, 0) + S(d, \pi, 0)$ rather than $\breve{A}_n^m$. In this case, the kernel-induced sound field interpolation method [23], which is a special case of the harmonic analysis-based method using omni-directional microphones [33], can be applied. The sound pressures at position $\boldsymbol{r}$ in the horizontal plane are then directly interpolated from the sound pressures recorded by the microphones in the planar distribution located at $\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_v, \cdots, \boldsymbol{r}_V$ without the truncated spherical harmonic order N as

$$S(\boldsymbol{r}) = \left((\boldsymbol{\Psi} + \lambda \boldsymbol{I}_V)^{-1} \boldsymbol{S}\right)^{\mathsf{T}} \boldsymbol{\kappa}(\boldsymbol{r}), \tag{16}$$

where

$$\boldsymbol{S} = [S(\boldsymbol{r}_1) \ \cdots \ S(\boldsymbol{r}_V)]^{\mathsf{T}}, \tag{17}$$

$$(\boldsymbol{\Psi})_{v,v'} = j_0(k||\boldsymbol{r}_v - \boldsymbol{r}_{v'}||), \tag{18}$$

$$\boldsymbol{\kappa}(\boldsymbol{r}) = [j_0(k||\boldsymbol{r} - \boldsymbol{r}_1||), \cdots, j_0(k||\boldsymbol{r} - \boldsymbol{r}_V||)]^{\mathsf{T}}, \tag{19}$$

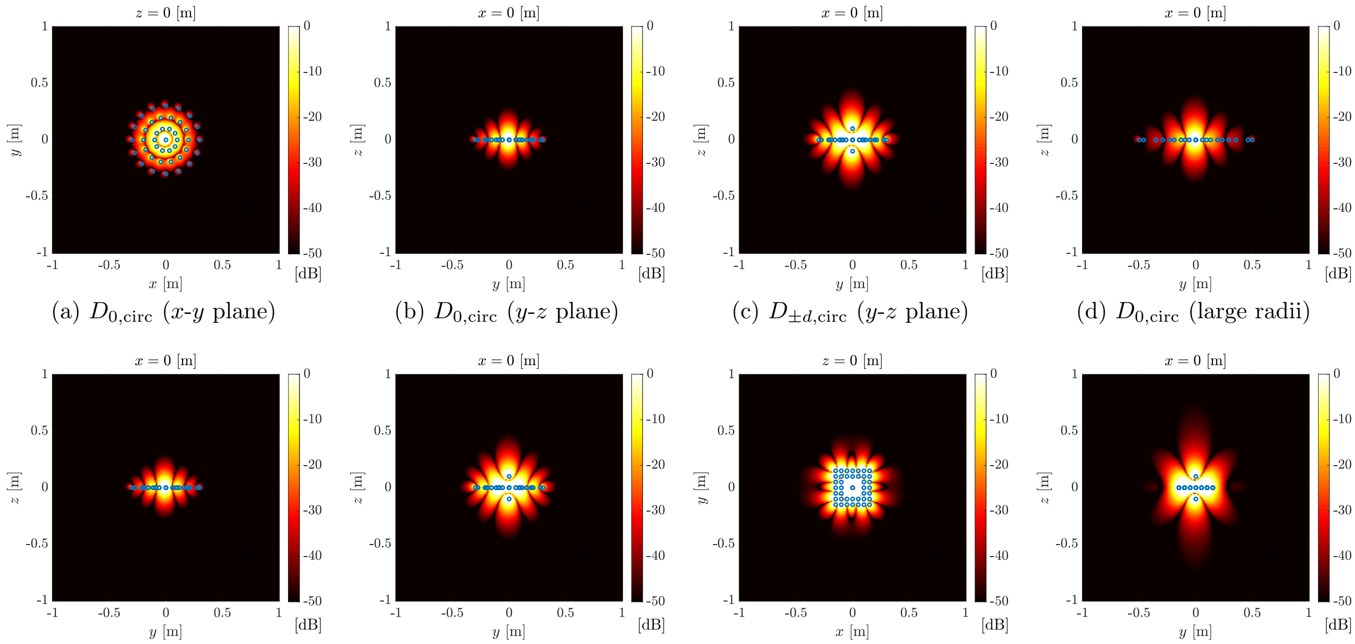
$\boldsymbol{I}_V$ is the identity matrix, and λ is a regularization parameter [23]. The residual when using an additional microphone on the origin is then given by

$$D_{0,\mathrm{dist}} = S(\boldsymbol{r}_0) - \left((\boldsymbol{\Psi} + \lambda \boldsymbol{I}_V)^{-1} \boldsymbol{S}\right)^{\mathsf{T}} \boldsymbol{\kappa}(\boldsymbol{r}_0), \tag{20}$$

where $\boldsymbol{r}_0 = (0, \pi/2, 0)$. Additionally, the residual when using the vertical microphone pair is derived as

$$D_{\pm d,\mathrm{dist}} = \sum_{i=1}^{2} \left\{ S(\boldsymbol{r}_i) - \left((\boldsymbol{\Psi} + \lambda \boldsymbol{I}_V)^{-1} \boldsymbol{S}\right)^{\mathsf{T}} \boldsymbol{\kappa}(\boldsymbol{r}_i) \right\}, \tag{21}$$

where $\boldsymbol{r}_1 = (d, 0, 0)$ and $\boldsymbol{r}_2 = (d, \pi, 0)$. Using (20) and (21), the proposed close-talking recording method with the microphones in a planar distribution on the horizontal plane can be realized without the truncated spherical harmonic order N, as shown in Fig. 2(c).



(a) $D_{0,\mathrm{circ}}$ (x-y plane) (b) $D_{0,\mathrm{circ}}$ (y-z plane) (c) $D_{\pm d,\mathrm{circ}}$ (y-z plane) (d) $D_{0,\mathrm{circ}}$ (large radii)

(e) $D_{0,\mathrm{dist}}$ (y-z plane) (f) $D_{\pm d,\mathrm{dist}}$ (y-z plane) (g) $D_{\pm d,\mathrm{dist}}$ (x-y plane) (h) $D_{\pm d,\mathrm{dist}}$ (y-z plane)

Fig. 3. Results for the residual pressure level $20 \log_{10} |D|$ used as the recording radial sensitivity for the proposed approaches. (a)-(f) were obtained using three layers of circular arrays; (a)-(b) $D_{0,\mathrm{circ}}$; (c) $D_{\pm d,\mathrm{circ}}$; (d) $D_{0,\mathrm{circ}}$ with large radii; (e) $D_{0,\mathrm{dist}}$; (f) $D_{\pm d,\mathrm{dist}}$; (g)-(h) $D_{\pm d,\mathrm{dist}}$ obtained using a rectangular array. Blue circles represent microphones.

4. COMPUTER SIMULATIONS

4.1. Simulation conditions

To evaluate the proposed close-talking recording methods, computer simulations were performed. A 3D free field was assumed in all simulations and the speed of sound c was 343.36 m/s. All microphones were omni-directional. In the experiments, three co-centered circular microphone arrays were introduced into the horizontal plane and their radii were set at 0.3, 0.2, and 0.1 m. The number of microphones in each circular array was 16, 14, and 10, respectively, as shown in Fig. 3(a)-(c), (e), and (f). Additionally, to estimate the effects of the radii of the arrays, arrays with radii of 0.5, 0.3, and 0.1 m and the same number of microphones were also introduced, as shown in Fig. 3(d). Therefore, the total numbers of microphones were 41 for $D_{0,\mathrm{circ}}$ and $D_{0,\mathrm{dist}}$, and 42 for $D_{\pm d,\mathrm{circ}}$ and $D_{\pm d,\mathrm{dist}}$. Additionally, a rectangular array containing 40 microphones in the horizontal plane was also introduced as shown in Fig. 3(g) and (h). In the method with the vertical microphone pair, d in (15) and (21) was set at 0.1 m. For the method based on multiple 2D cylindrical harmonic analyses, the maximum order N in (9) and (11) was set at 6. The regularization parameter λ in (20) and (21) was 10^{-5}.

To evaluate the recording radial sensitivity, a monopole source was arranged at $[x, y, z]^{\mathsf{T}}$ in Cartesian coordinates, where $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, and $-1 \leq z \leq 1$. Then, $20 \log_{10} |D|$ was evaluated using the sound pressures received via the microphones.

4.2. Simulation results

The results of the recording radial sensitivity for the frequency $f = 500$ Hz are plotted in Fig. 3. All results indicated that the proposed approaches based on both multiple 2D cylindrical harmonic analyses and kernel-induced sound field interpolation on the horizontal plane can realize close-talking recording inside the spherical boundary with the maximum radius of the arrays.

From Fig. 3(b), (c), (e) and (f), the recording radial sensitivities of the proposed methods based on both multiple 2D cylindrical harmonic analyses and kernel-induced sound field interpolation with multiple circular arrays were almost identical. By introducing a vertical microphone pair or arrays with larger radii, the recording radial sensitivity can be expanded compared with the methods using a microphone located at the origin, as shown in Fig. 3(c), (d) and (f). The results in Fig. 3(e) and (h) show that the proposed method based on kernel-induced sound field interpolation can be realized not only using multiple circular arrays but also using a rectangular array on the horizontal plane.

Consequently, the effectiveness of the proposed close-talking recording methods based on both multiple 2D cylindrical harmonic analyses and kernel-induced sound field interpolation on the horizontal plane was validated by the computer simulation results.

Future work will include detailed analysis of the relationships among the number of circular arrays, their radii, and the recording area shape, and experiments with practically implemented arrays.

5. CONCLUSIONS

This paper proposed a close-talking recording method with microphones in arbitrary planar distributions based on sound pressure interpolation. In the proposed method, sound pressures recorded using microphones located at the center of the array are interpolated using sound pressures recorded using the microphones in the planar distribution. Close-talking recording can then be realized as a residual response between the recorded and interpolated sound pressures. Computer simulations verified the effectiveness of the proposed close-talking recording method.

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