HORIZONTAL 3D SOUND FIELD RECORDING AND 2.5D SYNTHESIS WITH OMNI-DIRECTIONAL CIRCULAR ARRAYS

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ABSTRACT

Although 2.5D sound field synthesis with a circular loudspeaker array can be used in a 3D sound field, a 2D sound field, instead of a 3D sound field, is assumed for a sound field recording with a circular microphone array. This paper presents a horizontal 3D sound field recording and 2.5D synthesis method used in 3D sound fields with multiple co-centered omni-directional circular microphone arrays and a circular loudspeaker array without vertical derivative measurements. The spherical harmonic spectrums used for 2.5D synthesis are extracted from the recorded horizontal sound pressures and the driving function of a circular sound source is analytically derived based on 2.5D higher-order Ambisonics. The results of computer simulations indicate the effectiveness of the proposed method, in comparison with the conventional least squares-based pressure matching, and 2D cylindrical harmonic analysis-based approaches.

Index Terms— Horizontal 3D sound field recording, 2.5D sound field synthesis, circular array, spherical harmonic analysis, Ambisonics

1. INTRODUCTION

A 3D interior sound field can be recorded with a spherical microphone array mounted onto a spherical rigid baffle [1] and synthesized with a surrounding spherical loudspeaker array based on 3D higher-order Ambisonics (HOA) [2–4]. 3D HOA is based on the spherical harmonic expansion of a 3D sound field [5]. However, in 3D HOA, a large amount of microphones and loudspeakers more than $(N+1)^2$ elements are required for higher accuracy recording and synthesis with a maximum spherical harmonic order of N [6]. Therefore, it is not easy to implement these spherical arrays with a large number of microphones and loudspeakers in an actual environment.

To reduce the number of microphones and loudspeakers and realize practical applications, circular microphone and loudspeaker arrays, instead of spherical ones, are often introduced in the horizontal plane. When using 2D microphone and loudspeaker arrays, height-invariant 2D sound fields, both in recording and synthesis environments with height-invariant 2D line receivers and sources, must be considered. However, the actual environments are 3D sound fields, and the 3D omni-directional receivers and sources are often used in circular microphone and loudspeaker arrays, instead of 2D line receivers and sources.

Sound field synthesis methods with 2D linear and circular loudspeaker arrays in 3D sound fields are known as 2.5D sound field synthesis [4, 7–18]. Although there is a restriction that the correct sound pressures are only synthesized on a reference line or circle due to the dimension mismatch between the 2D array geometry and the 3D loudspeaker's propagation [15, 17], 2.5D sound field synthesis with 2D linear and circular loudspeaker arrays can be used in a 3D sound field.

On the other hand, a sound field recording with a circular array of 3D omni-directional microphones cannot be used in a 3D sound field because the recorded sound field is typically decomposed into 2D cylindrical harmonic spectrums that cannot represent the 3D sound field. Because of this problem, in existing sound field recording and synthesis methods with 2D circular and linear microphone and loudspeaker arrays, a 2D sound field is assumed in the primary recording field, and the recorded 2D sound field is synthesized by 2D [19–21] and 2.5D synthesis [8, 10, 12]. To avoid this problem, a spherical microphone array is introduced in the recording of a 3D sound field for 2.5D synthesis [16].

To record a 3D sound field with 2D planar microphone arrays, a method with multiple co-centered circular microphone arrays in the horizontal plane has been proposed [22–24].[1] By using these arrays, spherical harmonic spectrums $\breve{A}_n^m$ can be estimated and the recorded primary sound field can be reconstructed. However, this method must record not only the sound pressures, but also their vertical derivatives, because the recorded sound pressures can only estimate $\breve{A}_n^m$ for $n+|m|$ even, and their vertical derivatives can estimate those for $n+|m|$ odd. Therefore, this approach requires differential microphones or twice the number of omni-directional microphones for measuring both the sound pressures and their vertical derivatives.

To realize sound field recording and synthesis with the 2D microphone and loudspeaker arrays used in 3D sound fields, this study first considered that only a sound pressure measurement with multiple co-centered circular microphone arrays is sufficient for 2.5D synthesis with a circular loudspeaker array, because the 2.5D synthesis also includes only spherical harmonic spectrums $\breve{A}_n^m$ for $n+|m|$ even. This paper presents a horizontal 3D sound field recording and 2.5D synthesis method used in 3D sound fields with multiple co-centered omni-directional circular microphone arrays and a circular loudspeaker array. The processing flowchart of the proposed approach is illustrated in Fig. 1. In comparison with the conventional 2D sound field recording and 2.5D synthesis methods [8, 10, 12], the proposed approach can be entirely used in 3D sound fields. Additionally, the proposed method uses half the numbers of microphones in comparison with the original planar array method [22–24]. Moreover, the proposed method can estimate the spherical harmonic spectrums with a maximum order of N in the horizontal plane by using fewer than $(N+1)^2$ microphones used in a spherical array.

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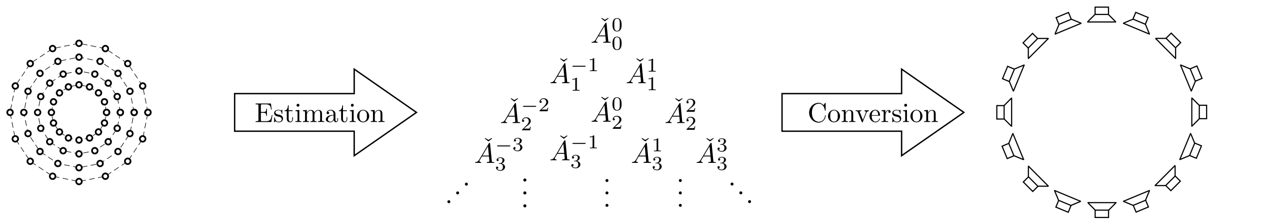
[1] This approach is extended to 3D sound field control with 2D planar loudspeaker arrays [25, 26] based on the acoustic source and receiver reciprocity.



(a) recording $S(r, \pi/2, \phi)$ (b) A_n^m for $n + |m|$ even (c) 2.5D synthesis with $D(\boldsymbol{r}_0)$

Fig. 1. Processing flowchart of proposed horizontal 3D sound field recording and 2.5D synthesis scheme: (a) recording horizontal 3D sound field $S(r, \pi/2, \phi)$ with multiple co-centered omni-directional circular arrays; (b) estimating spherical harmonic spectrums A_n^m for $n + |m|$ even from recorded horizontal sound pressures; (c) converting estimated spherical harmonic spectrums into driving function $D(\boldsymbol{r}_0)$ and synthesizing horizontal 3D sound field with a circular loudspeaker array based on 2.5D higher-order Ambisonics.

2. 3D SOUND FIELD IN HORIZONTAL PLANE

The interior expansion of a 3D sound field in a homogeneous region free of sources is given as:

$$S(r, \theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \breve{A}_n^m j_n(kr) Y_n^m(\theta, \phi), \tag{1}$$

where $\breve{A}_n^m$ and j_n are the spherical harmonic spectrums of the interior sound field, and the n-th order spherical Bessel function, k is the wavenumber [5];

$$Y_n^m(\theta, \phi) = \underbrace{\sqrt{\frac{(2n+1)}{4\pi} \frac{(n-|m|)!}{(n+|m|)!}} P_n^{|m|}(\cos\theta)}_{\mathcal{P}_n^{|m|}(\cos\theta)} e^{jm\phi} \tag{2}$$

is the spherical harmonics; $P_n^{|m|}$ is the associated Legendre function [27].

To consider a 3D sound field in the horizontal plane with $\theta = \pi/2$, (1) is represented as:

$$S(r, \pi/2, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \breve{A}_n^m j_n(kr) \mathcal{P}_n^{|m|}(0) e^{jm\phi}, \tag{3}$$

where only $\breve{A}_n^m$ for $n + |m|$ even is present because $\mathcal{P}_n^{|m|}(0) = 0$ for $n + |m|$ odd [22]. To obtain $\breve{A}_n^m$ for $n + |m|$ odd components, the vertical derivatives of $\mathcal{P}_n^{|m|}(0)$ are also required and can be measured by differential microphones or microphone pairs in the horizontal plane [22]. Thus, not only omni-directional, but also vertical differential microphones, are required to estimate the entire 3D sound fields [22]. However, as shown in (17), the driving function of the 2.5D synthesis with a circular loudspeaker array also includes $\mathcal{P}_n^{|m|}(0)$ and only has $\breve{A}_n^m$ for $n + |m|$ even components. Therefore, only omni-directional components are sufficient in the recording stage when the recording objective regards the 2.5D synthesis.

3. PROPOSED METHOD

3.1. Horizontal 3D sound field recording with multiple co-centered omni-directional circular microphone arrays

From (3), a continuous circular sound pressure distribution centered at the origin of the x-y plane ($\theta = 0$) is converted into 2D cylindrical harmonic spectrums [5] and represented as:

$$\mathring{S}_m(r) = \frac{1}{2\pi} \int_0^{2\pi} S(r, \pi/2, \phi) e^{-jm\phi} d\phi \tag{4}$$

$$= \sum_{n=|m|}^{\infty} \breve{A}_n^m j_n(kr) \mathcal{P}_n^{|m|}(0). \tag{5}$$

When the maximum spherical harmonic order is N and the radius of the circle is R_q, (5) is represented as

$$\mathring{S}_m(R_q) \simeq \sum_{n=|m|}^{N} \breve{A}_n^m j_n(kR_q) \mathcal{P}_n^{|m|}(0). \tag{6}$$

By introducing multiple radii ($q = 1, 2, \cdots, Q$) to (6), it can be represented in matrix form:

$$\mathring{\boldsymbol{S}}_m = \boldsymbol{U}_{|m|} \breve{\boldsymbol{A}}_m^{\text{even}}, \tag{7}$$

where

$$\mathring{\boldsymbol{S}}_m = \left[\mathring{S}_m(R_1),\ \mathring{S}_m(R_2),\ \cdots,\ \mathring{S}_m(R_Q)\right]^{\mathrm{T}}, \tag{8}$$

$$\boldsymbol{U}_{|m|} = \begin{bmatrix} U_{|m|}^{|m|}(kR_1) & U_{|m|+2}^{|m|}(kR_1) & \cdots & U_N^{|m|}(kR_1) \\ U_{|m|}^{|m|}(kR_2) & U_{|m|+2}^{|m|}(kR_2) & \cdots & U_N^{|m|}(kR_2) \\ \vdots & \vdots & \vdots & \vdots \\ U_{|m|}^{|m|}(kR_Q) & U_{|m|+2}^{|m|}(kR_Q) & \cdots & U_N^{|m|}(kR_Q) \end{bmatrix}, \tag{9}$$

$$U_n^{|m|}(kR) = j_n(kR) \mathcal{P}_n^{|m|}(0), \tag{10}$$

and

$$\breve{\boldsymbol{A}}_m^{\text{even}} = \left[\breve{A}_{|m|}^m,\ \breve{A}_{|m|+2}^m,\ \cdots,\ \breve{A}_N^m\right]^{\mathrm{T}}, \tag{11}$$

in the case where both N and m are either odd or even otherwise replace N in (9) and (11) by $N - 1$. Then, 3D spherical harmonic spectrums $\breve{A}_n^m$ for $n + |m|$ even can be obtained from multiple co-centered omni-directional circular sound pressure distributions [22]:

$$\breve{\boldsymbol{A}}_m^{\text{even}} = \left(\boldsymbol{U}_{|m|}^{\mathrm{T}} \boldsymbol{U}_{|m|}\right)^{-1} \boldsymbol{U}_{|m|}^{\mathrm{T}} \mathring{\boldsymbol{S}}_m, \tag{12}$$

as described in Figs 1(a) and (b).

3.2. 2.5D sound field synthesis with a circular loudspeaker array

A sound field synthesized by a continuous circular sound source distribution with a radius r_0 centered at the origin of the x-y plane is given as:

$$S(r,\theta,\phi) = \int_0^{2\pi} D(\boldsymbol{r}_0) G_{3\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}_0) r_0 d\phi_0, \tag{13}$$

where $G_{3\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}_0)$ is the transfer function from a sound source position $\boldsymbol{r}_0 = [r_0,\ \pi/2,\ \phi_0]^{\mathrm{T}}$ to a receiver position $\boldsymbol{r}$. Under the free-field assumption, $G_{3\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}_0)$ is the 3D free-field Green's function and given as [5]:

$$G_{3\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}_0) = \frac{e^{jk|\boldsymbol{r}-\boldsymbol{r}_0|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}_0|}. \tag{14}$$

When assuming an interior sound field, $G_{3\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}_0)$ can be represented by the spherical harmonic expansion and given as [5]:

$$G_{3\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}_0) = jk \sum_{n=0}^{\infty}\sum_{m=-n}^{n} j_n(kr) h_n(kr_0) Y_n^m(\theta,\phi) Y_n^m(\theta_0,\phi_0)^*, \tag{15}$$

where h_n is the n-th order spherical Hankel function of the first kind [5]. By applying the 2D cylindrical harmonic expansion to (13), the circular convolution theorem holds and (13) can be represented as:

$$\mathring{S}_m(r_{\mathrm{ref}}) = 2\pi \mathring{D}_m \mathring{G}_m(r_{\mathrm{ref}}, r_0), \tag{16}$$

where r_{ref} is the reference radius for 2.5D synthesis with a circular source. From (15), $\mathring{G}_m(r_{\mathrm{ref}}, r_0)$ is obtained as:

$$\mathring{G}_m(r_{\mathrm{ref}}, r_0) = jk \sum_{n=|m|}^{\infty} j_n(kr_{\mathrm{ref}}) h_n(kr_0) \mathcal{P}_n^{|m|}(0)^2. \tag{17}$$

This equation indicates that the driving function of 2.5D synthesis with a circular sound source also includes $\mathcal{P}_n^{|m|}(0)$, and only has $\breve{A}_n^m$ for $n+|m|$ even components. Then, the cylindrical harmonic spectrums of the driving function $\mathring{D}_m$ for synthesizing a horizontal sound field $\breve{A}_m^n$ for $n+|m|$ even estimated in (12) were analytically derived from (5), (16), and (17) [4]:

$$\mathring{D}_m = \frac{\sum_{n=|m|}^{\infty} \breve{A}_n^m j_n(kr_{\mathrm{ref}}) \mathcal{P}_n^{|m|}(0)}{2\pi \sum_{n=|m|}^{\infty} jk j_n(kr_{\mathrm{ref}}) h_n(kr_0) \mathcal{P}_n^{|m|}(0)^2}. \tag{18}$$

The driving function of 2.5D HOA with a reference radius $r_{\mathrm{ref}} = 0$ has been derived [4, 9, 13] and can synthesize a more accurate sound field in comparison with $r_{\mathrm{ref}} > 0$. To extend the driving function of the proposed method into that with $r_{\mathrm{ref}} = 0$, L'Hôpital's rule [28] is also applied in (18). The driving function with $r_{\mathrm{ref}} = 0$ is then obtained [4]:

$$\mathring{D}_m\Big|_{r_{\mathrm{ref}}=0} = \frac{\breve{A}_{|m|}^m}{2\pi jk h_{|m|}(kr_0) \mathcal{P}_{|m|}^{|m|}(0)}. \tag{19}$$

(19) is obviously simpler than (18) because there are no summation terms and it is thus calculated only from $\breve{A}_{|m|}^m$ components.

The continuous multiple co-centered omni-directional circular sound pressure distributions and a circular source are finally discretized into multiple co-centered omni-directional circular microphone arrays and a circular loudspeaker array [20]. Using the proposed approach, a horizontal 3D sound field recorded with multiple co-centered omni-directional circular microphone arrays can be synthesized with a circular loudspeaker array based on 2.5D HOA, as shown in Fig. 1.

4. COMPUTER SIMULATIONS

Computer simulations evaluated the proposed approach and compared it with the conventional least squares-based pressure matching (PM) method [29, 30] and 2D cylindrical harmonic analysis-based recording and 2.5D synthesis approach [10]. In all the simulations, a 3D free field was assumed and speed of sound c was 343.36 m/s.

In the recording stage, five co-centered omni-directional circular microphone arrays were introduced in the horizontal plane, and these radii were set to 0.4, 0.35, 0.3, 0.25, and 0.2 m, respectively. The target frequency band was up to 1 kHz. The maximum order was obtained as $N = \lceil ekR_{q,\max}/2 \rceil = 10$ [6] and the number of microphones on each circular array was calculated as 21, 19, 17, 15, and 11, according to an array design procedure provided in [22]. Then, the total number of microphones was 83, which is fewer than $(N+1)^2$ used in a spherical microphone array.

In the synthesis stage, the radius of a circular loudspeaker array was $r_0 = 0.75$ m and the number of loudspeakers was 21, according to maximum order $N = 10$. To evaluate the estimated spherical harmonic spectrums, the reconstructed sound field in the horizontal plane was calculated by (3) up to a maximum order of $N = 10$.

To compare the proposed method with the conventional 2D cylindrical harmonic analysis-based recording and 2.5D synthesis approach, the 2D sound field recording with multiple circular arrays [20] and the 2.5D HOA for 2D sound field synthesis [9, 13] were directly combined. The 2D cylindrical harmonic spectrums were estimated from the recorded sound field $\mathring{S}_m(R_q)$, as follows:

$$\mathring{A}_m = \frac{\mathring{S}_m(R_q)}{J_m(kR_q)}, \tag{20}$$

where J_m is the m-th order Bessel function, and R_q was selected from the radii of the multiple circular microphone arrays for the maximum value of $|J_m(kR_q)|$, so as to avoid forbidden frequencies [20, 31]. Then, the driving function of the 2.5D synthesis for the estimated 2D sound field was obtained [9, 13]:

$$\mathring{D}_{m,\mathrm{cyl}} = \frac{\mathring{A}_m J_m(kr_{\mathrm{ref}})}{2\pi \sum_{n=|m|}^{\infty} jk j_n(kr_{\mathrm{ref}}) h_n(kr_0) \mathcal{P}_n^{|m|}(0)^2}. \tag{21}$$

Similar to (19), the L'Hôpital's rule was also applied in (21) [9, 13]:

$$\mathring{D}_{m,\mathrm{cyl}}\Big|_{r_{\mathrm{ref}}=0} = \frac{\mathring{A}_m \mathrm{sgn}(m)^{|m|}(2|m|+1)!!}{2^{|m|}|m|!2\pi jk h_{|m|}(kr_0) \mathcal{P}_{|m|}^{|m|}(0)^2}, \tag{22}$$

where sgn is the sign function. Additionally, to evaluate the estimated 2D cylindrical harmonic spectrums, the reconstructed sound field in the horizontal plane was calculated as [5]:

$$S(r,\pi/2,\phi)) = \sum_{m=-N}^{N} J_m(kr) \mathring{A}_m e^{jm\phi}. \tag{23}$$

In all the simulations, r_{ref} was set to 0 m and the driving functions of (19) and (22) were used. In the PM method, the truncated singular value decomposition [32] with 40 dB was introduced to calculate the stable driving signals. The maximum order of the 2.5D HOA for the 2D sound field synthesis was also obtained and used up to $N = 10$. The two sound source locations in the recording sound field were $\boldsymbol{s} = [2,\ 2,\ 0]^{\mathrm{T}}$ and $[2,\ 2,\ 2]^{\mathrm{T}}$ in Cartesian coordinates.

To estimate the reconstructed and synthesized sound field, the reconstruction / synthesis error at position $\boldsymbol{r}$ is defined as

$$E(\boldsymbol{r}) = 10\log_{10}\frac{|S_{\mathrm{org}}(\boldsymbol{r}) - S_{\mathrm{rec/syn}}(\boldsymbol{r})|^2}{|S_{\mathrm{org}}(\boldsymbol{r})|^2}, \tag{24}$$

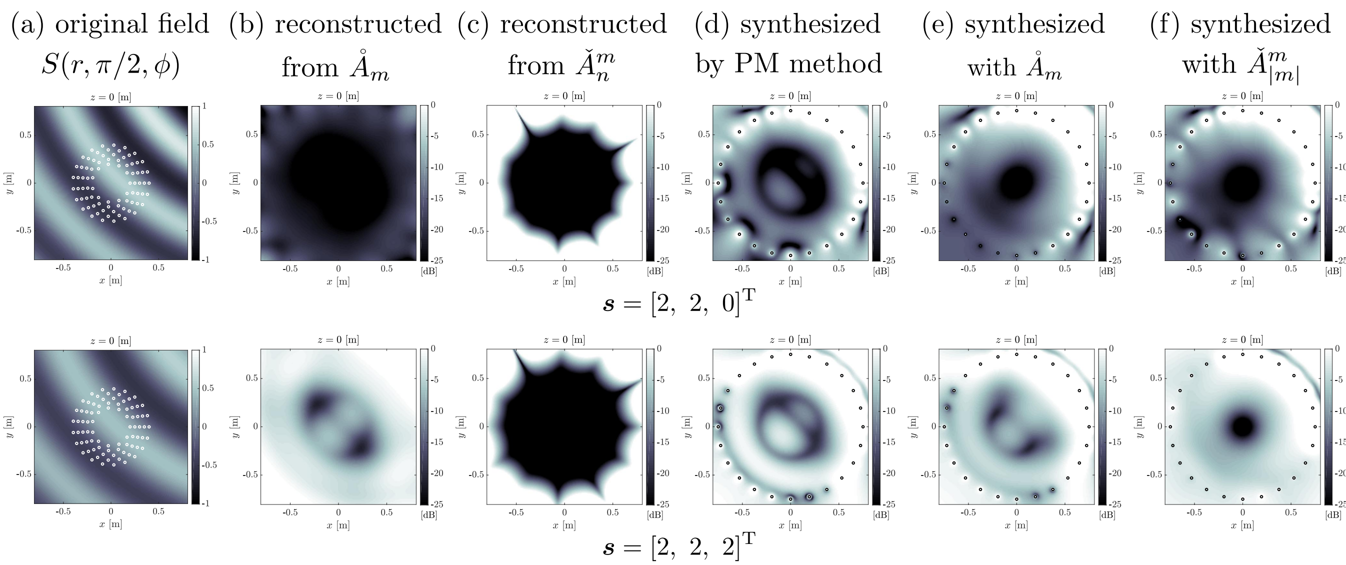


Fig. 2. Results of (a) original sound field; (b)-(f) reconstruction and synthesis errors in horizontal plane for two primary sound source locations of $\boldsymbol{s} = [2,\ 2,\ 0]^{\mathrm{T}}$ and $[2,\ 2,\ 2]^{\mathrm{T}}$ and frequency $f = 500$ Hz (b) reconstructed from estimated $\mathring{A}_m$; (c) reconstructed from estimated $\breve{A}_n^m$; (d) synthesized by pressure matching (PM) method; (e) synthesized by 2.5D HOA for 2D sound field $\mathring{A}_m$; (f) synthesized by 2.5D HOA for horizontal 3D sound field $\breve{A}_{|m|}^m$. White circles in (a) and black circles in (d)-(f) represent microphones and loudspeakers, respectively.

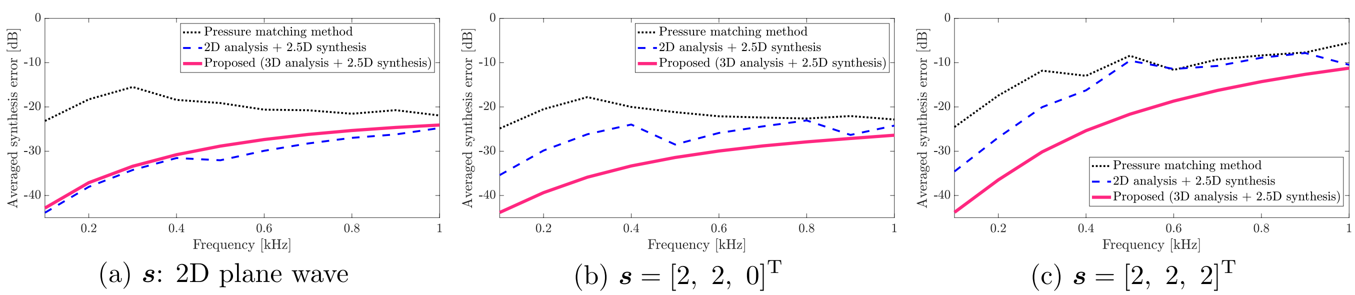


Fig. 3. Results of averaged synthesis error in horizontal plane for $r \leq 0.2$ m and 100 Hz $\leq f \leq 1$ kHz: (a) primary recording sound field is 2D plane wave; (b) source position in primary sound field is $\boldsymbol{s} = [2,\ 2,\ 0]^{\mathrm{T}}$; (c) source position in primary sound field is $\boldsymbol{s} = [2,\ 2,\ 2]^{\mathrm{T}}$.

where $S_{\mathrm{org}}(\boldsymbol{r})$ and $S_{\mathrm{rec/syn}}(\boldsymbol{r})$ are the original and reconstructed / synthesized sound pressures at position $\boldsymbol{r}$, respectively. Additionally, the averaged synthesis error for $r \leq 0.2$ m was also calculated to evaluate the synthesis accuracy around the array center, including an additional case where the primary recording sound field was a height-invariant 2D plane wave with unity amplitude.

Figures 2 and 3 present the results of the reconstruction and synthesis errors for a frequency of $f = 500$ Hz, and the averaged synthesis error for 100 Hz $\leq f \leq 1$ kHz.

These results suggest that the proposed method can more accurately synthesize the horizontal 3D sound fields recorded by multiple circular microphone arrays compared with the conventional PM and 2D analysis-based approaches, particularly in the case where the primary sound source is not located in the horizontal plane. This is because the proposed approach can correctly estimate 3D spherical harmonic spectrums $\breve{A}_n^m$ for $n + |m|$ even, and extract only $\breve{A}_{|m|}^m$ components. When the primary field is a 2D plane wave, the conventional 2D analysis-based method can be used to achieve high accuracy. However, the PM and 2D analysis-based approaches cannot correctly synthesize the horizontal 3D sound field, particularly in the case of $\boldsymbol{s} = [2,\ 2,\ 2]^{\mathrm{T}}$ because the 3D sound fields include not only horizontal but also vertical components. However, these methods cannot extract the horizontal components from the 3D sound fields.

Consequently, the proposed approach can effectively record a horizontal 3D sound field and synthesize it using omni-directional circular microphone and loudspeaker arrays without vertical derivative measurements. Additionally, the proposed method can estimate the spherical harmonic spectrums with a maximum order of N using fewer than $(N+1)^2$ microphones used in a spherical array.

5. CONCLUSIONS

This paper proposed a horizontal 3D sound field recording and 2.5D synthesis method with multiple co-centered circular microphone arrays and a circular loudspeaker array without vertical derivative measurements. The spherical harmonic spectrums used for 2.5D synthesis were extracted from the recorded horizontal sound pressures and the driving function of a circular sound source was analytically derived. The results of computer simulations validated the effectiveness of the proposed method compared with the conventional PM and 2D cylindrical harmonic analysis-based approaches.

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