2.5D HIGHER ORDER AMBISONICS FOR A SOUND FIELD DESCRIBED BY ANGULAR SPECTRUM COEFFICIENTS

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ABSTRACT

This paper derives an analytical solution to convert sound field representation from the angular spectrum to the circular harmonics expansion. A sound field is decomposed to plane waves by the spatial Fourier transform and represented by the angular spectrum. A plane wave is also described by the circular harmonics expansion. In the proposed formulation, these two representations are integrated and a sound field can be represented by the circular harmonics expansion with the angular spectrum coefficients. For actual implementations, the driving function of a circular sound source for 2.5D higher order Ambisonics is analytically derived from a sound field described by the angular spectrum. The results of the computer simulations show that the proposed method with a circular loudspeaker array can reproduce a sound field recorded by a linear microphone array with appropriate accuracy around the center of the circular array.

Index Terms— Angular spectrum, circular harmonics, spherical harmonics, sound field reproduction, Ambisonics

1. INTRODUCTION

Sound field synthesis is an important acoustic communication technique and several methods have been investigated in the last decades.

Higher order Ambisonics (HOA) [1–7] has been investigated for a sound field synthesis technique which can synthesize sound waves coming from all directions. In HOA, spherical and circular microphone arrays [8–10] are introduced to record sound fields and spherical and circular loudspeaker arrays are used in reproduction stage. A sound field in HOA is described by the spherical and cylindrical harmonics expansions which are derived from the spatial Fourier transform in spherical and cylindrical coordinates [11].

For synthesizing sound waves from a half space, wave field synthesis (WFS) [12–15] and spectral division method (SDM) [6, 16–18] have been proposed. In WFS and SDM, planar and linear arrays of microphones and loudspeakers [15,17,18] are introduced for sound field recording and reproduction, respectively. A sound field in SDM is described by the angular spectrum which is derived from the spatial Fourier transform in Cartesian coordinates [11].

Since these methods provide analytical solutions based on the spatial Fourier transform, sound field representations should be the same formats between recording and reproduction stages. For this reason, in these methods, the configurations of receivers and secondary sources should be spherical/circular and planar/linear arrangements. In HOA, several decoding methods for hemispherical [19], multiple circular [20] and irregular [21, 22] loudspeaker layouts have been investigated.

In contrast, there is no representation consistency between recording and reproduction stages in pressure-matching based least squares (LS) [23–26], lasso [27] and matching pursuit [28] approaches. An HOA encoding method with LS based plane wave decomposition for irregular microphone arrays also has been proposed [29]. These numerical approaches, however, are quite unstable because the acoustic inverse problem is very ill-conditioned [11,30].

For these reasons, it is highly important to improve the flexibility of analytical approaches for sound field recording and reproduction. An analytical approach for WFS with truncated linear loudspeaker arrays to reproduce a sound field described by the spherical harmonics expansion coefficients has been provided [31]. For sound field recording by using spherical and circular microphone arrays, an analytical method of converting between cylindrical and spherical harmonics representations of sound fields has been proposed [32].

This paper provides an analytical solution for converting sound field representation from the angular spectrum to the circular harmonics expansion. The proposed formulation enables a sound field captured by a linear microphone array to be reproduced by a circular loudspeaker array based on HOA.

For actual implementations, sound field reproduction systems are frequently simplified to be reproduced in the horizontal plane. Then the secondary sources are arranged on a line or a circle rather than a plane or a sphere. In actual implementations, monopole sources instead of line sources are usually employed for secondary sources. Such approaches are called 2.5D sound field synthesis [3–5, 7, 16, 33]. The driving functions for both 2D and 2.5D HOA [3–5, 7], therefore, are analytically derived from a sound field described by the angular spectrum.

2. CONVERTING SOUND FIELD REPRESENTATION FROM ANGULAR SPECTRUM TO CIRCULAR HARMONICS EXPANSION

Spherical coordinates relative to Cartesian coordinates for both the spatial and wavenumber $k(=\omega/c)$ domains are defined in Fig. 1(a) and (b), respectively. $\omega = 2\pi f$ is the angular frequency, f denotes the temporal frequency and c is the speed of sound.

Assuming that all sound waves are come from a half space $y < 0$ on the x-y plane ($z = k_z = 0$ and $\theta = \theta_k = \pi/2$). In this case, the trace wavenumber in the y direction is $k_y = \sqrt{k^2 - k_x^2} \geq 0$ and a sound field $P(x, y, \omega)$ is then represented as [11,16]

$$P(x, y, \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{P}(k_x, \omega) e^{-j\sqrt{k^2-k_x^2}y} e^{-jk_x x} dk_x, \quad (1)$$

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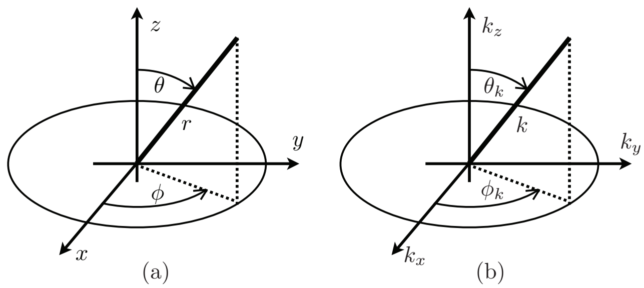


Fig. 1. Definition of spherical coordinates relative to Cartesian coordinates. (a): spatial domain and (b): wavenumber domain.

where $\tilde{P}(k_x, \omega)$ is the angular spectrum. (1) indicates that a sound field is decomposed to each plane wave propagating to ϕ_k with each weight coefficient $\tilde{P}(k_x, \omega)$. When a far field is assumed, $P(x, y, \omega)$ only contains the propagation wave components [11] and (1) can be represented as

$$P(x, y, \omega) \simeq \frac{1}{2\pi} \int_{-k}^{k} \tilde{P}(k_x, \omega) e^{-j\sqrt{k^2-k_x^2}y} e^{-jk_x x} dk_x. \quad (2)$$

From Fig. 1, $P(x, y, \omega)$ is expressed as a sum of plane waves propagating to the direction ϕ_k from π to 0 with the weight coefficients $\tilde{P}(k_x, \omega)$.

For converting the representation of (2) from the angular spectrum to the circular harmonics expansion, the coordinate system of (2) is transformed from Cartesian coordinates to cylindrical coordinates, and represented as

$$\begin{aligned} P(r, \phi, \omega) &= \frac{1}{2\pi} \int_{\pi}^{0} \tilde{P}(\phi_k, \omega) e^{-jkr(\sin(\phi)\sin(\phi_k)+\cos(\phi)\cos(\phi_k))} \\ &\qquad \cdot ((k\cos(\phi_k))' \, d\phi_k \\ &= \frac{k}{2\pi} \int_{0}^{\pi} \tilde{P}(\phi_k, \omega) e^{-jkr\cos(\phi-\phi_k)} \sin(\phi_k) d\phi_k, \quad (3) \end{aligned}$$

where $x = r\cos(\phi)$, $y = r\sin(\phi)$, $k_x = k\cos(\phi_k)$ and the integration by substitution $dk_x \to d\phi_k$ is applied.

At the same time, a plane wave propagating to the direction ϕ_k is also represented by the circular harmonics expansion [11] and given as

$$e^{-jkr\cos(\phi-\phi_k)} = \sum_{m=-\infty}^{\infty} (-j)^m J_m(kr) e^{jm(\phi-\phi_k)}, \quad (4)$$

where J_m is the m-th order Bessel function. From (3) and (4), the circular harmonics representation of $P(r, \phi, \omega)$ is analytically obtained as

$$\begin{aligned} &\mathring{P}_{2\mathrm{D},m}(r, \omega) \\ &\quad = \frac{k(-j)^m J_m(kr)}{2\pi} \int_{0}^{\pi} \tilde{P}(\phi_k, \omega) \sin(\phi_k) e^{-jm\phi_k} d\phi_k. \quad (5) \end{aligned}$$

To derive the driving function of 2.5D HOA in the next session, the 2.5D spherical harmonics expansion of a plane wave [3,4] is also introduced as

$$\begin{aligned} &e^{-jkr\cos(\phi-\phi_k)} \\ &\quad = \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} j_n(kr) \underbrace{4\pi(-j)^n Y_n^m(\pi/2, \phi_k)^*}_{\mathring{P}_n^m(\phi_k, \omega)} Y_n^m(\pi/2, \phi), \end{aligned} \quad (6)$$

where j_n and Y_n^m are the n-th order spherical Bessel function and the m-th order spherical harmonics of the n-th degree [11], respectively. From (3) and (6), the 2.5D spherical harmonics representation of $P(r, \phi, \omega)$ is also analytically derived as

$$\mathring{P}_{2.5\mathrm{D},m}(r, \omega) = \frac{k}{2\pi} \int_{0}^{\pi} \tilde{P}(\phi_k, \omega) \mathring{\Psi}_m(r, \phi_k, \omega) \sin(\phi_k) d\phi_k, \quad (7)$$

where

$$\begin{aligned} &\mathring{\Psi}_m(r, \phi_k, \omega) \\ &\quad = \sum_{n=|m|}^{\infty} (-j)^n j_n(kr)(2n+1) \frac{(n-m)!}{(n+m)!} P_n^m(0)^2 e^{-jm\phi_k}, \quad (8) \end{aligned}$$

and P_n^m is the m-th order associated Legendre polynomial of the n-th degree [11].

As a result, a sound field described by the angular spectrum can be represented by the circular harmonics expansion with the angular spectrum coefficients as (5) and (7).

3. 2.5D HOA FOR A SOUND FIELD DESCRIBED BY ANGULAR SPECTRUM COEFFICIENTS

3.1. Analytical formulation

By using the 2D circular and 2.5D spherical harmonics expansion coefficients of a sound field derived in (5) and (7), the driving functions of 2D and 2.5D HOA are analytically derived in this section.

The angular spectrum representation of a sound field can be obtained by a continuous linear receiver. When the receiver is located along with the x-axis, the recorded sound pressure $P(x, 0, \omega)$ is transformed to the angular spectrum coefficients by the spatial Fourier transform [11] along with x and given as

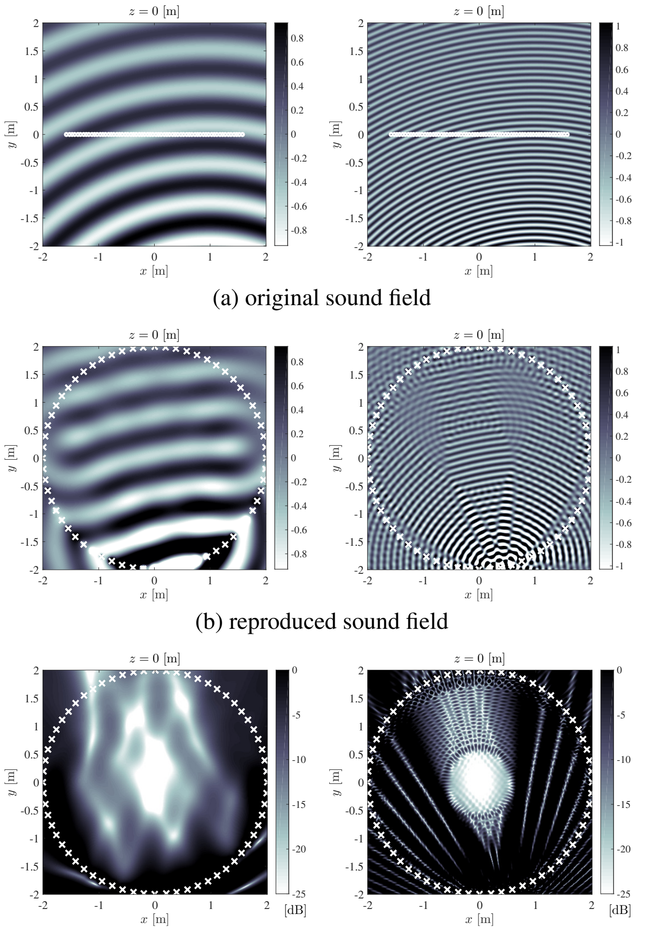
$$\begin{aligned} \tilde{P}(\phi_k, \omega) = \tilde{P}(k_x, \omega) &= \int_{-\infty}^{\infty} P(x, 0, \omega) e^{jk_x x} dx \\ &= \int_{-\infty}^{\infty} P(x, 0, \omega) e^{jk\cos(\phi_k)x} dx. \quad (9) \end{aligned}$$

A sound field synthesized by a continuous circular sound source distribution with radius r_0 centered at the origin on the x-y plane is given as

$$P(r, \phi, \omega) = \int_{0}^{2\pi} D(\boldsymbol{r}_0, \omega) G(\boldsymbol{r}, \boldsymbol{r}_0, \omega) r_0 d\phi_0, \quad (10)$$

where $G(\boldsymbol{r}, \boldsymbol{r}_0, \omega)$ is the transfer function from a sound source position $\boldsymbol{r}_0$ to a receiver position $\boldsymbol{r}$. Under the free-field assumption, $G(\boldsymbol{r}, \boldsymbol{r}_0, \omega)$ is the two-dimensional free-field Green's function for 2D HOA and the three-dimensional free-field Green's function given as

$$G_{3\mathrm{D}}(\boldsymbol{r}, \boldsymbol{r}_0, \omega) = \frac{e^{-jk|\boldsymbol{r}-\boldsymbol{r}_0|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}_0|}, \quad (11)$$



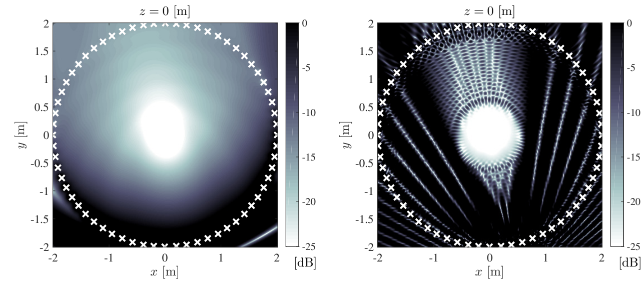
(a) original sound field

(b) reproduced sound field

(c) reproduction error $E(\boldsymbol{r})$ defined in (18)

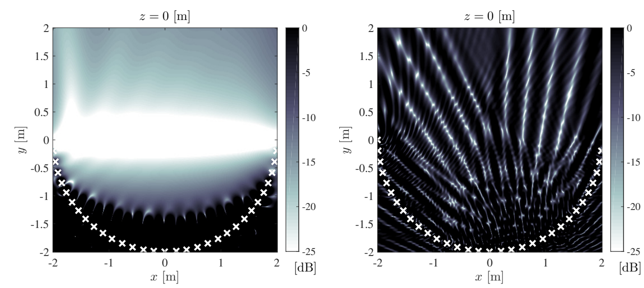
$f = 500$ Hz $f = 3$ kHz

Fig. 2. Results of proposed 2.5D HOA reproduction of sound field recorded by linear array of 64 microphones. White circles along with x-axis are 64 microphones and white crosses are 64 loudspeakers.



$f = 500$ Hz $f = 3$ kHz

Fig. 3. Synthesis error $E(\boldsymbol{r})$ defined in (18) of ideal point source synthesis by conventional 2.5D HOA using circular array of 64 loudspeakers.



$f = 500$ Hz $f = 3$ kHz

Fig. 4. Reproduction error $E(\boldsymbol{r})$ defined in (18) of conventional LS based reproduction of sound field recorded by linear array of 64 microphones using semicircular array of 32 loudspeakers.

for 2.5D HOA, respectively [11].

Just as in [3, 4], when the spatial Fourier series expansion [11] is applied to (10), the circular convolution theorem holds and the driving function of a circular sound source is directly derived as

$$\mathring{D}_m(r_0, r, \omega) = \frac{\mathring{P}_m(r, \omega)}{2\pi r_0 \mathring{G}_m(r, r_0, \omega)}. \tag{12}$$

In this case, the reproduction area is assumed to be inside a circular source and $r < r_0$ is considered.

In 2D HOA,

$$\mathring{G}_{\mathrm{2D}}(r < r_0, \omega) = -\frac{j}{4} J_m(kr) H_m^{(2)}(kr_0), \tag{13}$$

derived in [11] and the driving function of 2D HOA is analytically derived from (5), (12) and (13) as

$$\mathring{D}_{\mathrm{2D},m}(r_0, \omega) = \frac{k(-j)^{m+1}}{\pi^2 r_0 H_m^{(2)}(kr_0)} \int_0^\pi \tilde{P}(\phi_k, \omega) \sin(\phi_k) e^{-jm\phi_k} d\phi_k, \tag{14}$$

where $H_m^{(2)}$ is the m-th order Hankel function of the second kind. In (14), $J_m(kr)$ is cancelled and the driving function of 2D HOA is independent of r.

Compared to 2D HOA, on the other hand, the driving function of 2.5D HOA is dependent on r [3, 4]. In [3, 4], for avoiding forbidden frequencies where $j_n(kr) = 0$ [34], r is set to 0 by using the de l'Hospital's rule and the driving function of 2.5D HOA for $r = 0$ is analytically derived as

$$\mathring{D}_{\mathrm{2.5D},m}(r = 0, r_0, \omega) = \frac{1}{2\pi r_0} \cdot \frac{\check{P}_{|m|}^m(\omega)}{-jkh_{|m|}^{(2)}(kr_0) Y_{|m|}^m(\pi/2, 0)^*}, \tag{15}$$

where $h_n^{(2)}$ denotes the n-th order spherical Hankel function of the second kind [11].

Finally, by integrating (3), (6) and (15), the driving function of the proposed 2.5D HOA is analytically derived as

$$\begin{aligned}
&\mathring{D}_{\mathrm{2.5D},m}(r_0, \omega) \\
&= \frac{1}{2\pi r_0} \cdot \frac{1}{-jkh_{|m|}^{(2)}(kr_0) Y_{|m|}^m(\pi/2, 0)^*} \\
&\quad \cdot \frac{k}{2\pi} \int_0^\pi \tilde{P}(\phi_k, \omega) 4\pi (-j)^{|m|} Y_{|m|}^m(\pi/2, \phi_k)^* \sin(\phi_k) d\phi_k \\
&= \frac{(-j)^{|m|-1}}{\pi r_0 h_{|m|}^{(2)}(kr_0)} \int_0^\pi \tilde{P}(\phi_k, \omega) \sin(\phi_k) e^{-jm\phi_k} d\phi_k.
\end{aligned} \tag{16}$$

3.2. Actual implementation

For an actual implementation, a circular loudspeaker array instead of a continuous circular source is used and (16) must be discretized. When the number of loudspeakers is L_{sp}, the order m of the spatial Fourier series in (16) can be calculated up to $M = \lfloor (L_{\mathrm{sp}} - 1)/2 \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function. Finally, the driving signal of each loudspeaker at ϕ_l in the temporal frequency domain is obtained as

$$D(r_0, \phi_l, \omega) = \sum_{m=-M}^{M} \mathring{D}_m(r_0, \omega) e^{jm\phi_l}, \quad l = 1, 2, \cdots L_{\mathrm{sp}}. \tag{17}$$

4. COMPUTER SIMULATIONS

Computer simulations were performed to evaluate the proposed 2.5D HOA for a sound field recorded by a linear microphone array.

In all the simulations, a three-dimensional free field was assumed. The speed of sound c was 343.36 m/s. A circular loudspeaker array with radius $r_0 = 2.0$ m was centered at the origin and located on the x-y plane. The number of loudspeakers L_{sp} was 64. The sound pressures produced by a point source located at $\boldsymbol{x}_{\mathrm{s}} = [1,\ -5,\ 0]^{\mathrm{T}}$ were recorded by a linear array of $L_{\mathrm{mic}} = 64$ microphones centered at the origin and along with the x-axis as shown in Fig. 2(a). The distance between the adjacent microphones of the linear array was $\Delta x = 0.05$ m and the length of the array was 3.2 m. The spatial Nyquist frequency of the linear array was about 3.4 kHz.

In the proposed method, the recorded sound field was transformed to the angular spectrum coefficients by the discrete Fourier transform as the discretized and truncated representation of (9) where dx was discretized as Δx and the infinite integral was truncated as $-L_{\mathrm{mic}}\Delta x/2 \leq x \leq L_{\mathrm{mic}}\Delta x/2$. The driving signals of the circular loudspeaker array were also numerically calculated from the discretized representation of (16) where $d\phi_k$ was equiangularly discretized as $\Delta\phi_k = \pi/L_{\mathrm{mic}}$ and (17) with $M = 31$. The reproduced sound field was calculated from the discretized representation of (10).

To estimate the reproduced sound field, the reproduction error at position $\boldsymbol{r}$ was defined as

$$E(\boldsymbol{r}, \omega) = 10 \log_{10} \frac{|P_{\mathrm{org}}(\boldsymbol{r}, \omega) - P_{\mathrm{syn}}(\boldsymbol{r}, \omega)|^2}{|P_{\mathrm{org}}(\boldsymbol{r}, \omega)|^2}, \tag{18}$$

where $P_{\mathrm{org}}(\boldsymbol{r}, \omega)$ and $P_{\mathrm{syn}}(\boldsymbol{r}, \omega)$ were the original and reproduced sound pressures at position $\boldsymbol{r}$, respectively.

The results of the proposed method were compared with that of ideal point source synthesis by the conventional 2.5D HOA [3, 4] as a reference and the conventional pressure-matching based LS method [23, 24].

In ideal point source synthesis, the driving signals for synthesizing a sound field radiated from a point source located at $\boldsymbol{r}_{\mathrm{s}} = [r_{\mathrm{s}},\ \theta_{\mathrm{s}},\ \phi_{\mathrm{s}}]^{\mathrm{T}}$ were calculated from (15) where

$$\breve{P}^m_{|m|}(\boldsymbol{r}_{\mathrm{s}}, \omega) = -jkh^{(2)}_{|m|}(kr_{\mathrm{s}}) Y^m_{|m|}(\theta_{\mathrm{s}}, \phi_{\mathrm{s}})^*, \tag{19}$$

derived in [4] was introduced.

When a sound field recorded by a linear microphone array is reproduced by the LS method using a circular loudspeaker array inside the circle, it is obvious that only the semicircular portion is available for not only matching the sound pressures at the control points but also reproducing the wave front in the circle. In the simulations, a semicircular array of 32 loudspeakers was then used as shown in Fig. 4.

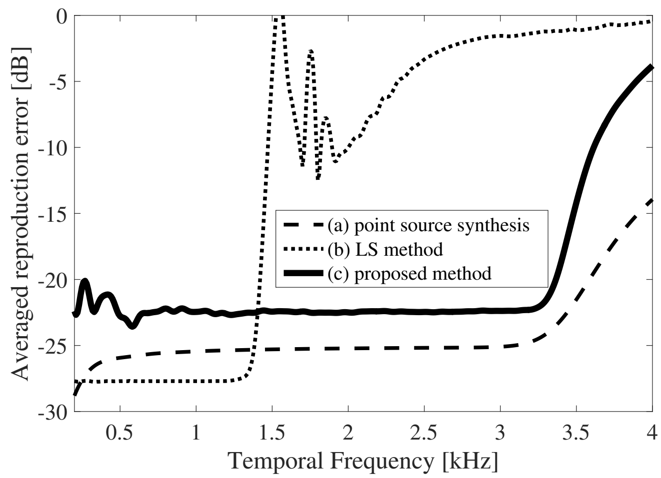


Fig. 5. Results of averaged reproduction error within $r \leq 0.5$ m for (a) ideal point source synthesis using circular array of 64 loudspeakers (reference), (b) LS method using semicircular array of 32 loudspeakers, and (c) proposed method using circular array of 64 loudspeakers.

Fig. 2 shows the results of the reproduced sound field and the reproduction error by the proposed method at $f = 500$ and 3000 Hz with the original sound field. Figs. 3 and 4 also show the results of the reproduction error by ideal point source synthesis and the conventional LS method, respectively. In addition, the results of the averaged reproduction error $\int_{r \leq 0.5} E(\boldsymbol{r}, \omega) d\boldsymbol{r}$ was also calculated up to $f = 4$ kHz and plotted in Fig. 5.

These results suggest that the conventional LS method can only control sound field at low frequencies because only 32 loudspeakers were used and the spatial Nyquist frequency of the LS method is about 1.7 kHz which corresponds to half as much as $c/(2\Delta x) \approx$ 3.4 kHz. The mode-matching based 2.5D HOA including the proposed method, on the other hand, can control sound field around the center of the array over a wide frequency range. Compared with the results of ideal point source synthesis as a reference, the proposed method can reproduce a sound field recorded by a linear microphone array with appropriate accuracy. The reproduction accuracy of the proposed method is slightly degraded because of the discretization of (9) and (16) and the truncation of (9). Detailed analysis and discussion are required as future work.

Consequently, the proposed analytical formulation is effective for HOA based synthesis of a sound field described by the angular spectrum coefficients.

5. CONCLUSION

This paper proposed an analytical method to convert sound field representation from the angular spectrum to the circular harmonics expansion. In the proposed method, these two representations are integrated and a sound field can be represented by the circular harmonics expansion with the angular spectrum coefficients. The driving functions of a circular sound source for both 2D and 2.5D HOA were analytically derived from a sound field described by the angular spectrum. The results of the computer simulations with 2.5D HOA using a circular loudspeaker array showed that the proposed formulation can reproduce a sound field recorded by a linear microphone array with appropriate accuracy around the center of the circular array.

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