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Analytical approach to 2.5D sound field control using a circular double-layer array of fixed-directivity loudspeakers

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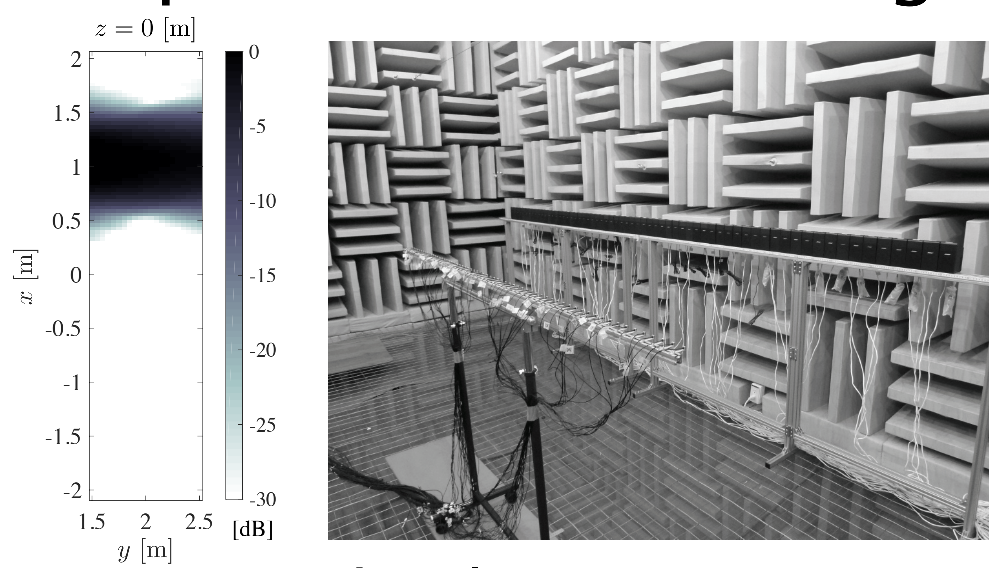
Outline

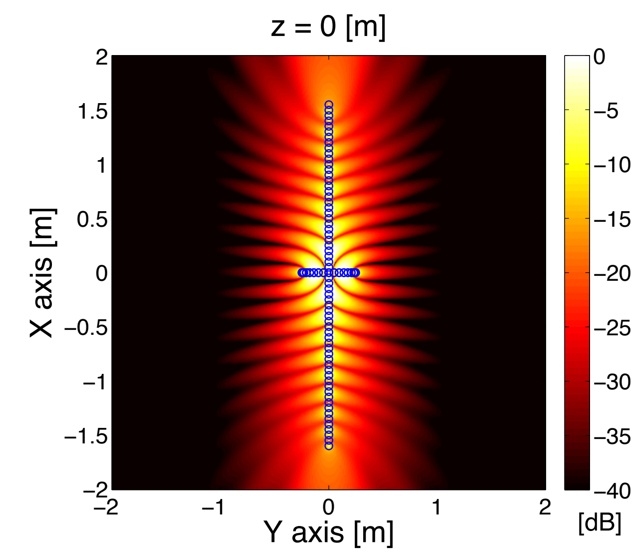
- Introduction
  - Research focus: Sound field control considering acoustically dark zones
  - Motivation: Undesired propagation free sound field synthesis
  - 2D Interior and exterior sound fields
  - What is "2.5D" sound field control?
- Conventional approaches and their limitations
  - Analytical 2D Interior and exterior sound field control
  - 2.5D sound field control with a circular double-layer loudspeaker array
- Proposed method
  - Analytical 2.5D approach with a circular double-layer of source
  - Interior field control with reference radius $r_{\mathrm{i}} = 0$
  - Computer simulations
- Concluding remarks

Research focus

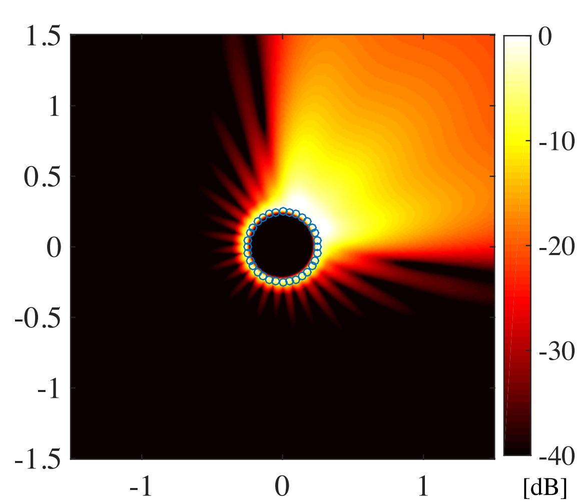
- Analytical approaches to sound field control with loudspeakers considering acoustically bright and dark zones



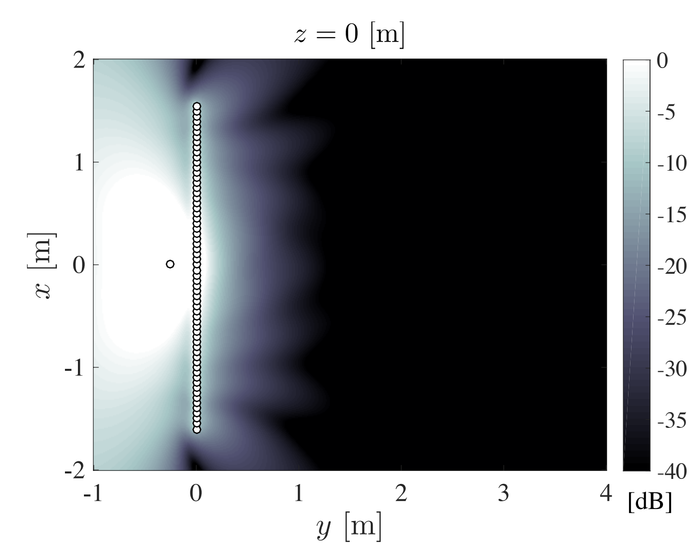
with a linear array
ICASSP 2014 and JASA (in press)



with a linear and circular array combination
ICASSP 2015



with a baffled circular array
WASPAA 2015

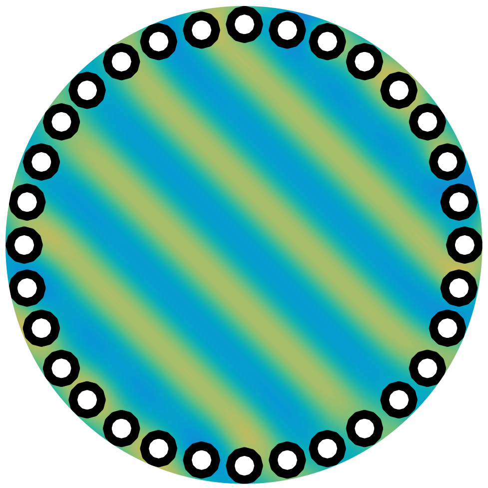


with a linear array and a point source
JIHMSP (in press)

Motivation

- Undesired propagation free sound field synthesis
  - Typical sound field synthesis methods

e.g. Wave field synthesis (WFS), Higher order Ambisonics (HOA)



Simple 2D HOA

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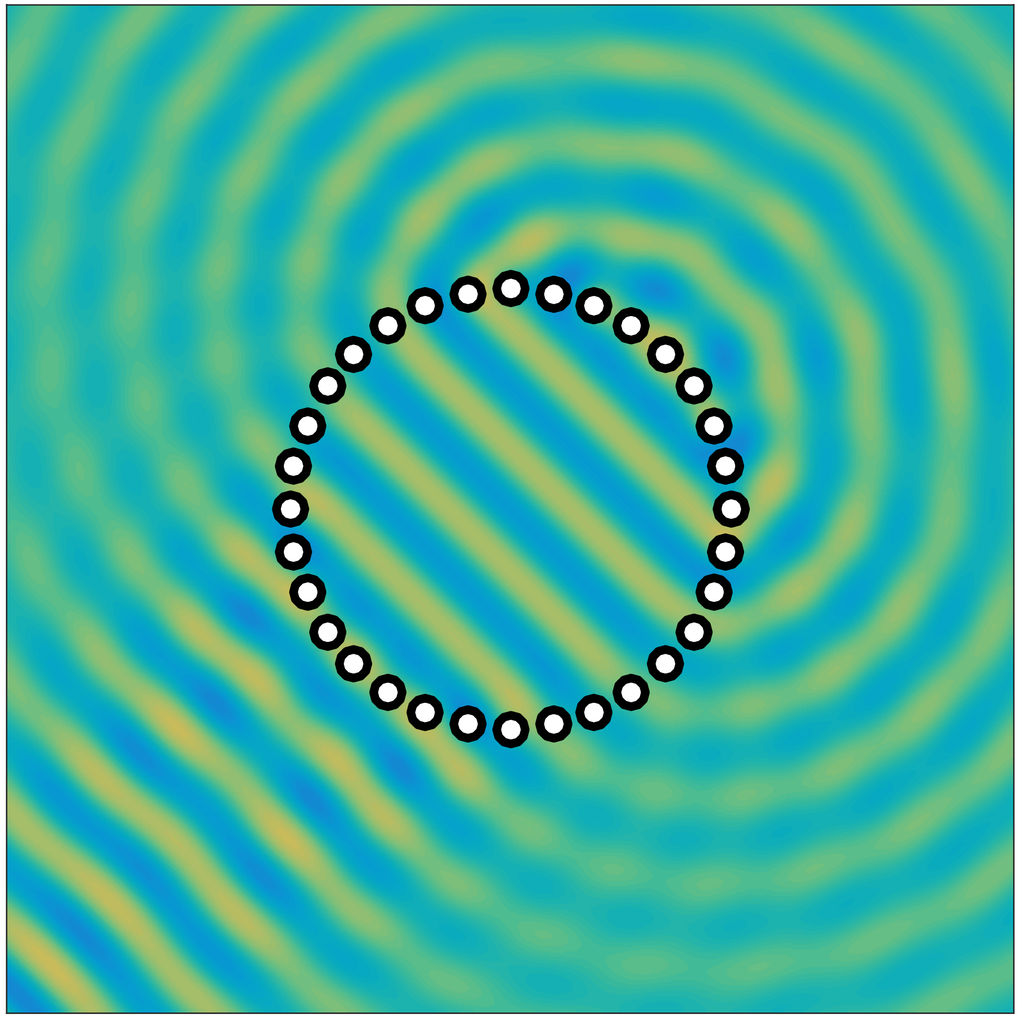

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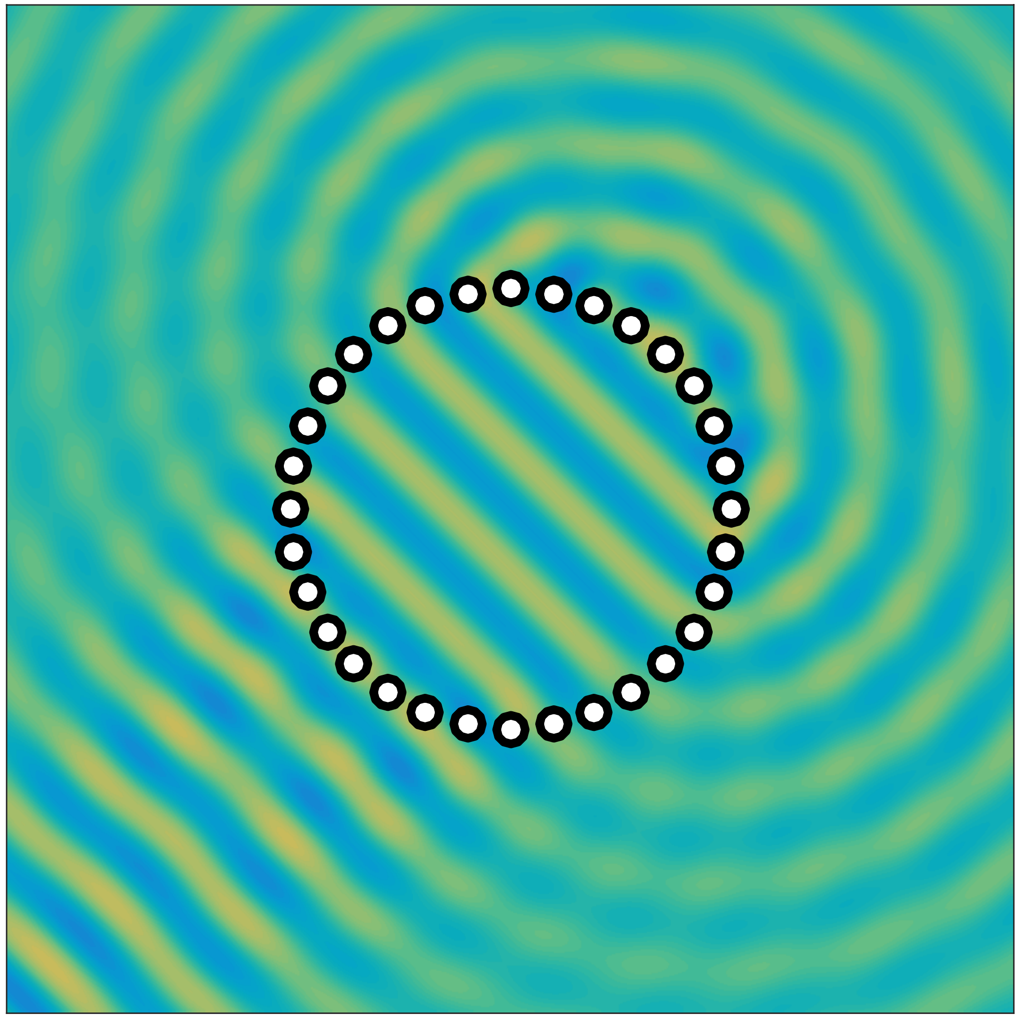

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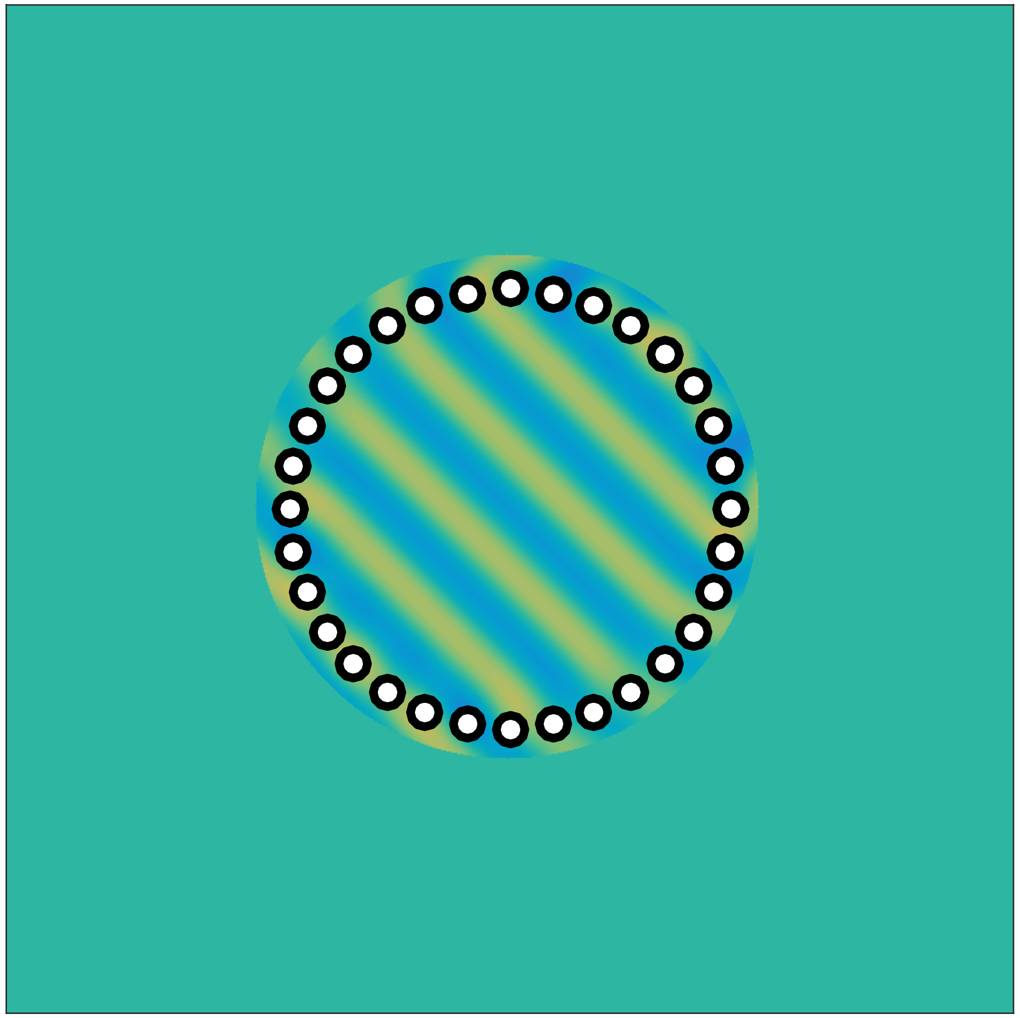


Undesired propagation free sound field synthesis

2D interior and exterior sound fields

Fourier series expansion of 2D sound fields

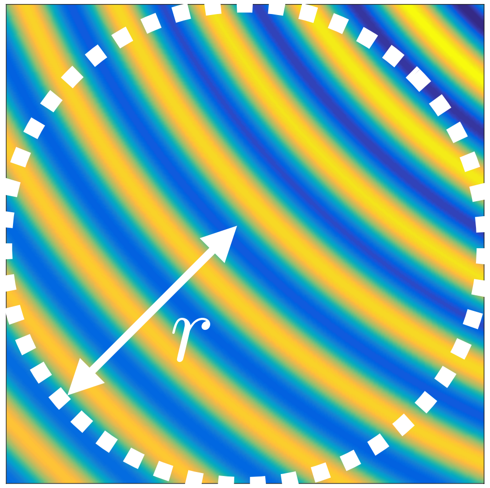
Interior sound field decomposition

$$S(r,\phi) = \sum_{m=-\infty}^{\infty} \mathring{A}_m J_m(kr) e^{jm\phi}$$

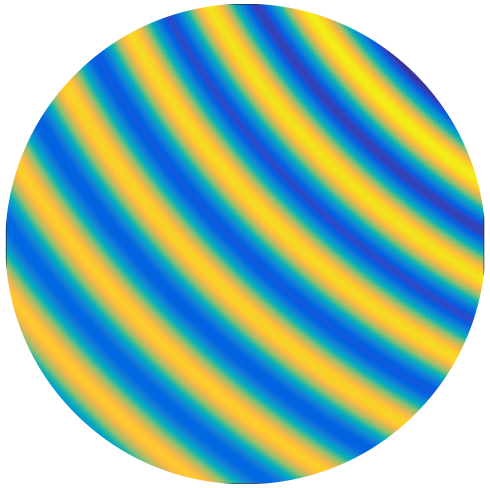
S : sound pressure

$\mathring{A}_m$: interior Fourier series coefficients

J_m : Bessel function



Original



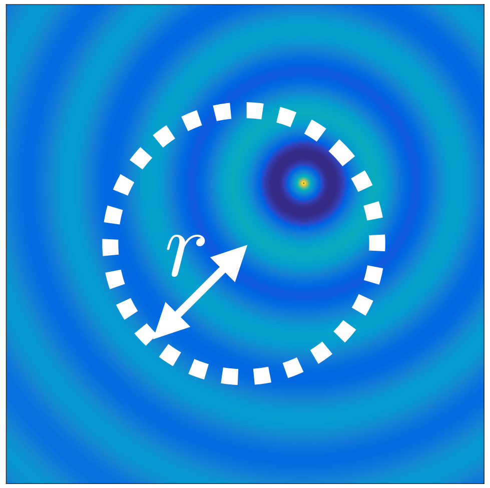
Interior

Exterior sound field decomposition

$$S(r,\phi) = \sum_{m=-\infty}^{\infty} \mathring{B}_m H_m(kr) e^{jm\phi}$$

$\mathring{B}_m$: exterior Fourier series coefficients

H_m : Hankel function of 1st kind



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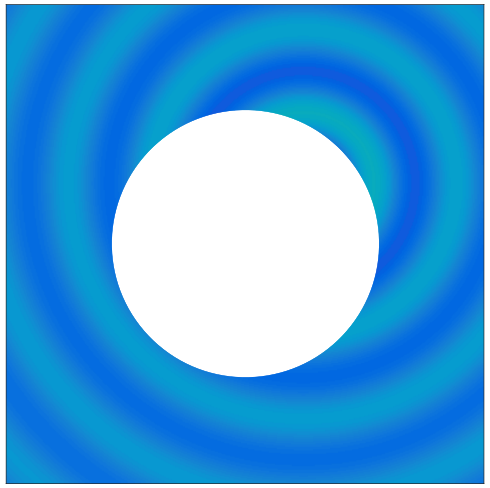
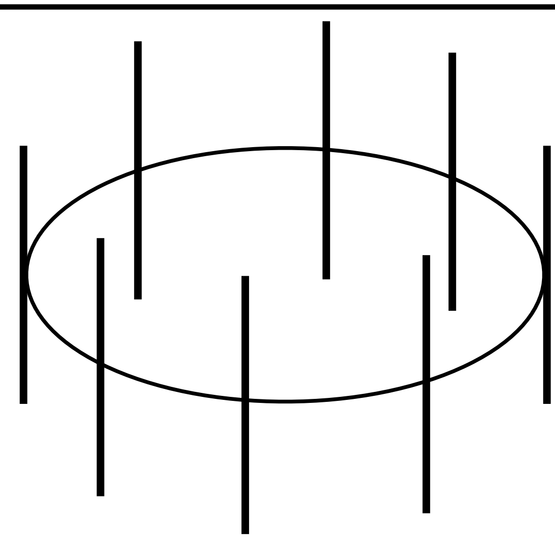
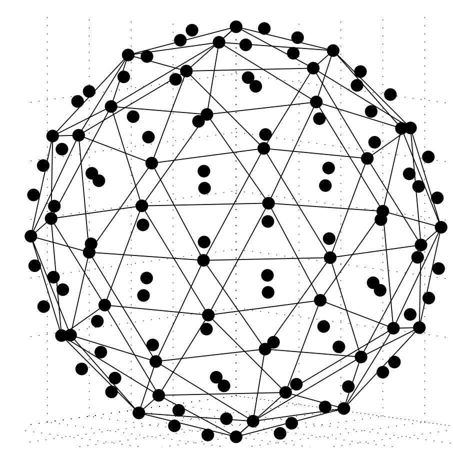


Exterior

What is “2.5D” sound field control?

- 2.5D sound field control
  - Horizontal sound field control using 2D array configuration with 3D propagation sound sources

	2D synthesis	3D synthesis
Array configuration	 2D	 3D
Sound source	Line source	Point source
Radiation pattern	Cylindrical	Spherical
Implementation	Difficult	Difficult

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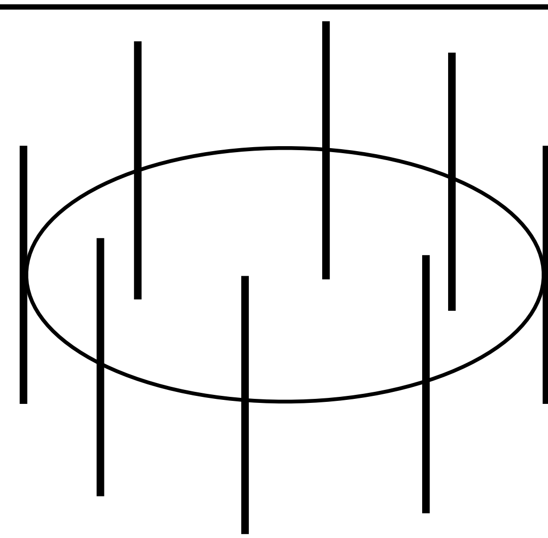
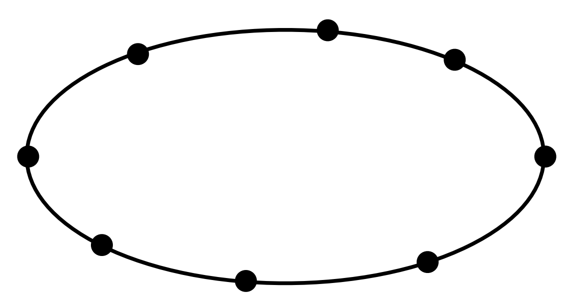
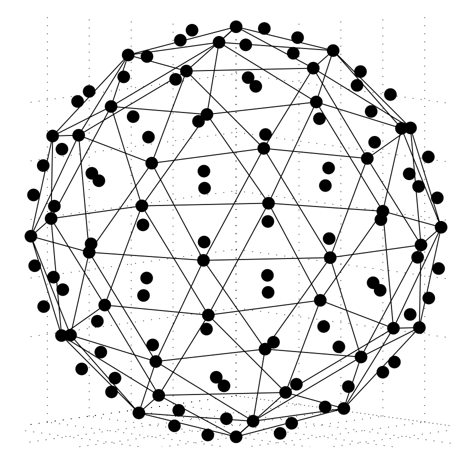
Array configuration	 2D	 2D	 3D
Sound source	Line source	Point source	Point source
Radiation pattern	Cylindrical	Spherical	Spherical
Implementation	Difficult	Easy	Difficult

Conventional approaches

- Analytical 2D interior and exterior sound field control
  - No undesired sound pressures are propagated to exterior field
    - Two types of driving functions can control interior and exterior field
  - Limitations
    - 1st order line sources are required

(Poletti et al., JASA, 2011.)

- 2.5D Sound field control with a circular double-layer array of fixed directivity loudspeakers

  (Chang et al., JASA, 2012.)
  - Least squares (LS) approach

    $$\mathbf{D}(k) = \mathbf{G}(k)^{+}\mathbf{P}(k)$$
  - Problem
    - $\mathbf{G}(k)$ is quite ill-conditioned
  - Limitation
    - Regularization schemes are required

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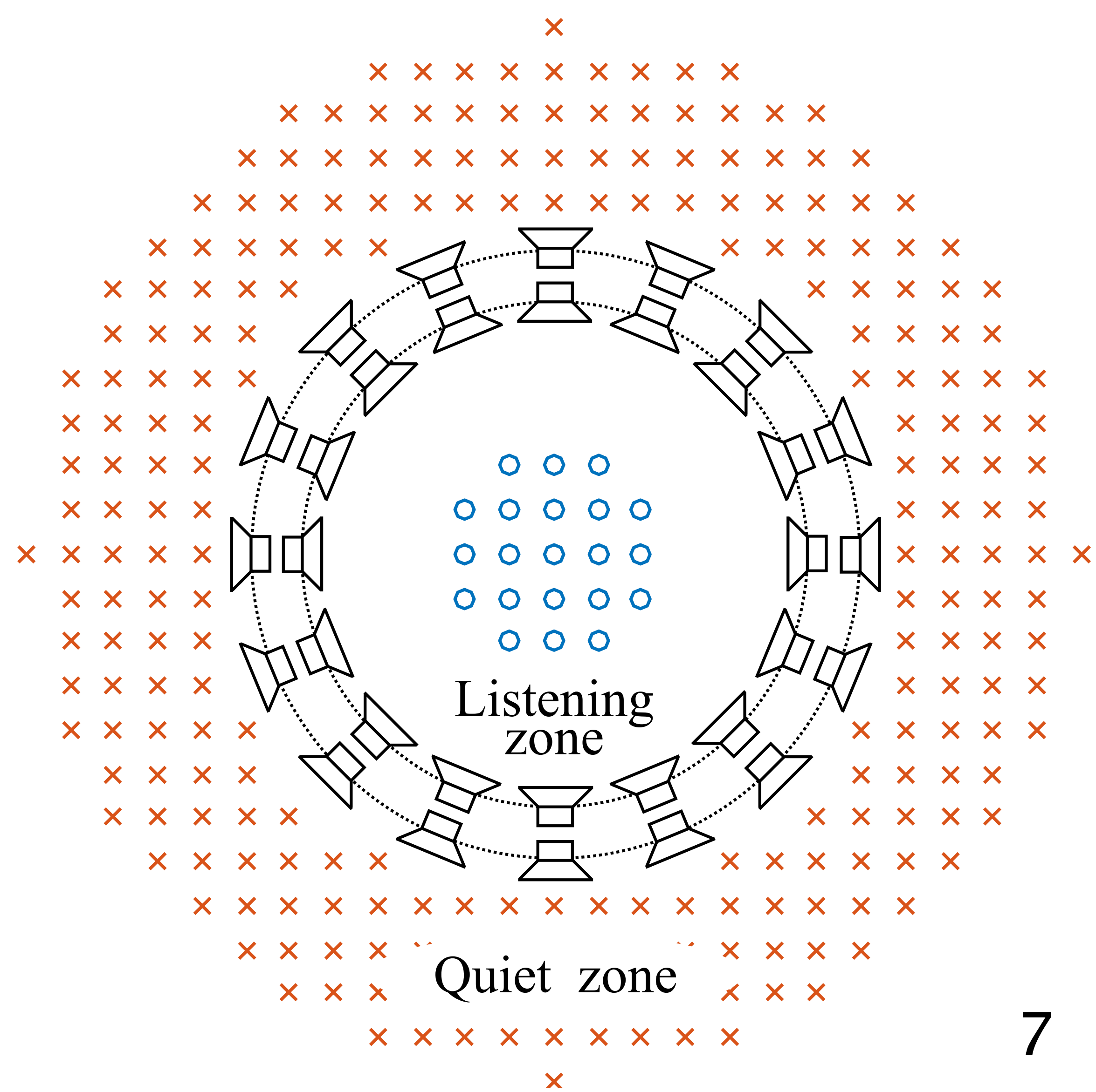
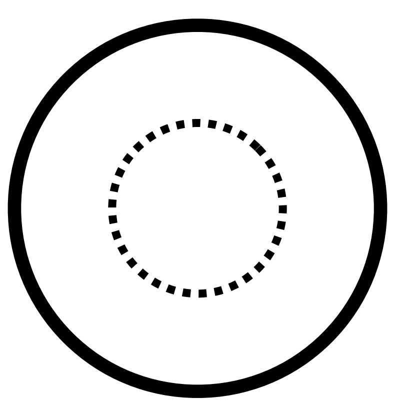


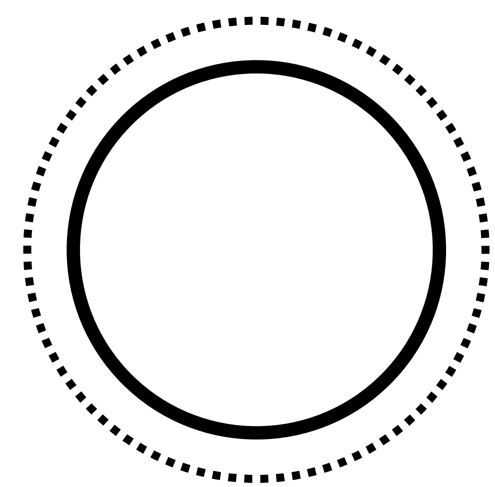
Proposed approach

- Regularization-free analytical approach to 2.5D sound field control with a circular double-layer array of fixed directivity loudspeakers
  - 2.5D interior and exterior HOA integration
    - Interior HOA (Ahrens et al., ICASSP 2008, Zhang et al. IWAENC 2014.)
    - Exterior HOA (Okamoto, WASPAA 2015.)

$$S(r,\theta,\phi) = \int_0^{2\pi} D(\phi_0) G_{\mathrm{3D}}(\boldsymbol{r}, \boldsymbol{r}_0) d\phi_0 \quad \xrightarrow{\mathcal{F}_\phi} \quad \mathring{D}_m = \frac{\mathring{S}_m(r)}{2\pi \mathring{G}_{\mathrm{3D},m}(r, r_0)}$$



Interior 2.5D HOA



Exterior 2.5D HOA

  - Using fixed directivity loudspeakers
    - 2.5D spherical harmonic representation of transfer functions

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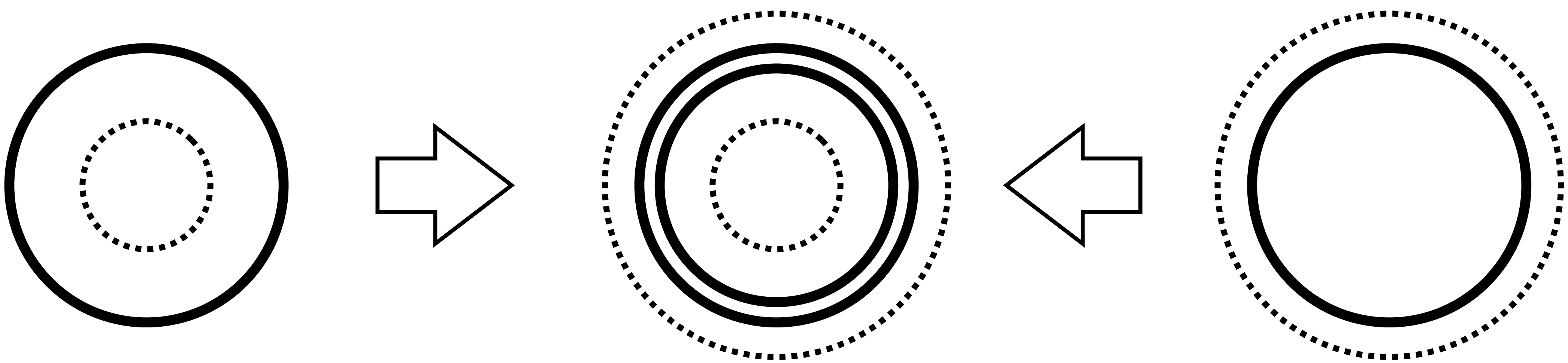


Interior 2.5D HOA | Proposed method | Exterior 2.5D HOA

  - Using fixed directivity loudspeakers
    - 2.5D spherical harmonic representation of transfer functions

Proposed formulation

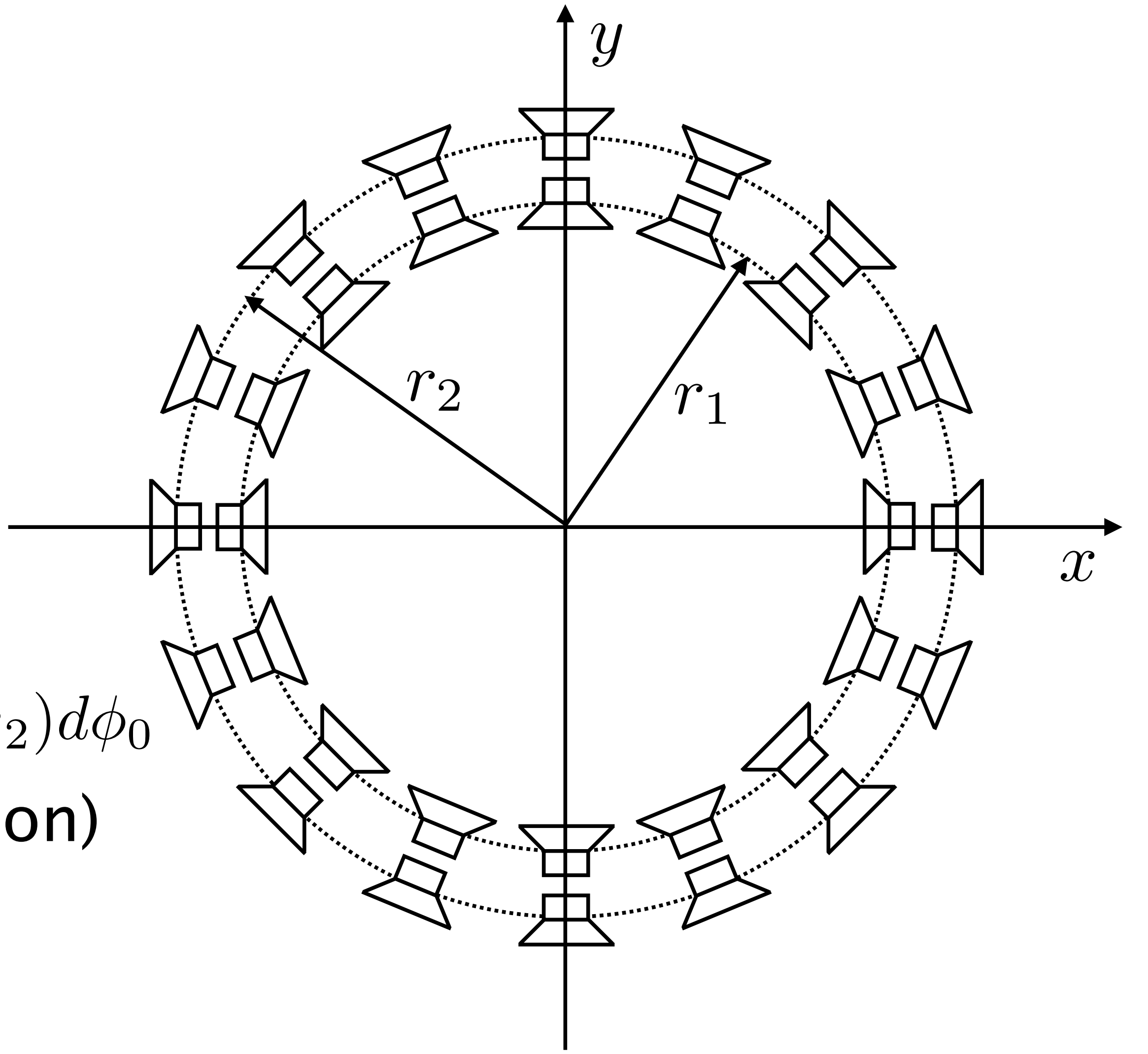
Sound field produced by a circular double layer source

S : produced sound field

D_1 and D_2 : driving functions

T_1 : transfer functions (inward)

T_2 : transfer functions (outward)



$$S(r, \theta, \phi) = \int_0^{2\pi} D_1(\phi_0) T_1(\boldsymbol{r}, \boldsymbol{r}_1) + D_2(\phi_0) T_2(\boldsymbol{r}, \boldsymbol{r}_2) d\phi_0$$

(Convolution)

Fourier series expansion in the horizontal plane

$$\mathring{S}_m(r) = 2\pi \left\{ \mathring{D}_{m,1} \mathring{T}_{m,1}(r, r_1) + \mathring{D}_{m,2} \mathring{T}_{m,2}(r, r_2) \right\}$$

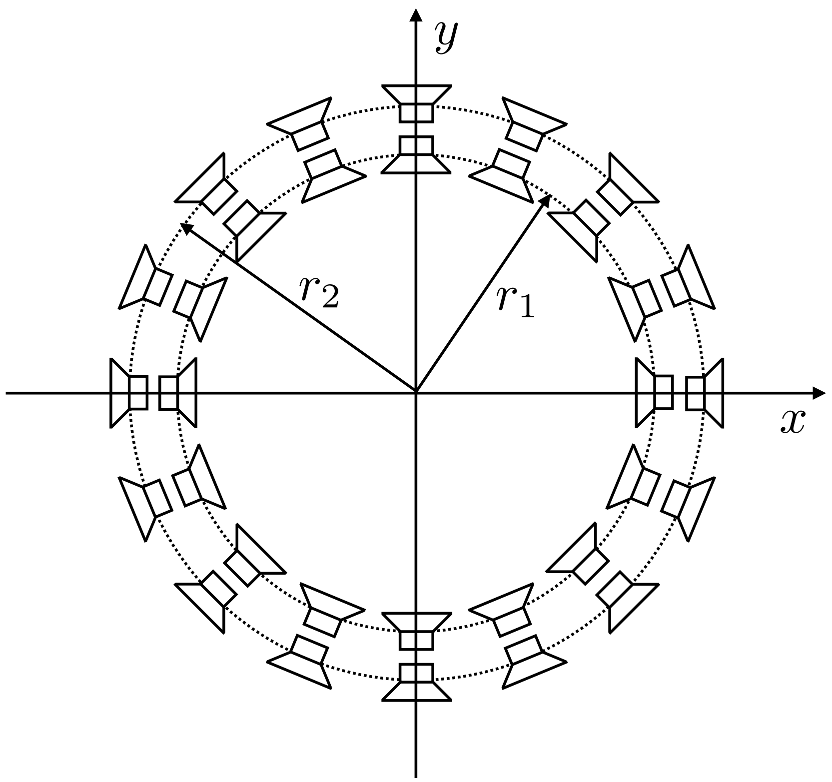
(Multiplication)

Using fixed directivity of loudspeakers

- 2.5D spherical harmonic representation of transfer functions
  - Transfer functions in the temporal frequency domain

$$T_1(\boldsymbol{r}, \boldsymbol{r}_1) = \frac{e^{jk|\boldsymbol{r}-\boldsymbol{r}_1|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}_1|}\left\{a - (1-a)\left[1 + \frac{j}{k|\boldsymbol{r}-\boldsymbol{r}_1|}\right]\cos\alpha\right\}$$

$$T_2(\boldsymbol{r}, \boldsymbol{r}_2) = \frac{e^{jk|\boldsymbol{r}-\boldsymbol{r}_2|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}_2|}\left\{a + (1-a)\left[1 + \frac{j}{k|\boldsymbol{r}-\boldsymbol{r}_2|}\right]\cos\alpha\right\}$$



  - 2.5D spherical harmonic representation

$$\mathring{T}_{m,1}(r_<, r_1) = jk \sum_{n=|m|}^{\infty} j_n(kr) h_{n,1}(a, r_1) Q_n^m P_n^{|m|}(0)^2$$

$$\mathring{T}_{m,2}(r_<, r_2) = jk \sum_{n=|m|}^{\infty} j_n(kr) h_{n,2}(a, r_2) Q_n^m P_n^{|m|}(0)^2$$

$$\mathring{T}_{m,1}(r_>, r_1) = jk \sum_{n=|m|}^{\infty} h_n(kr) j_{n,1}(a, r_1) Q_n^m P_n^{|m|}(0)^2$$

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$$h_{n,1}(a, r_1) = a h_n(kr_1) - j(1-a) h_n'(kr_1)$$
$$h_{n,2}(a, r_2) = a h_n(kr_2) + j(1-a) h_n'(kr_2)$$
$$j_{n,1}(a, r_1) = a j_n(kr_1) - j(1-a) j_n'(kr_1)$$
$$j_{n,2}(a, r_2) = a j_n(kr_2) + j(1-a) j_n'(kr_2)$$
$$Q_n^m = \frac{2n+1}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}.$$

Proposed interior and exterior control

- Interior and exterior sound field control in the horizontal plane
  - Driving function of 2.5D HOA depends on radius of control circle
    - Reference radius must be set

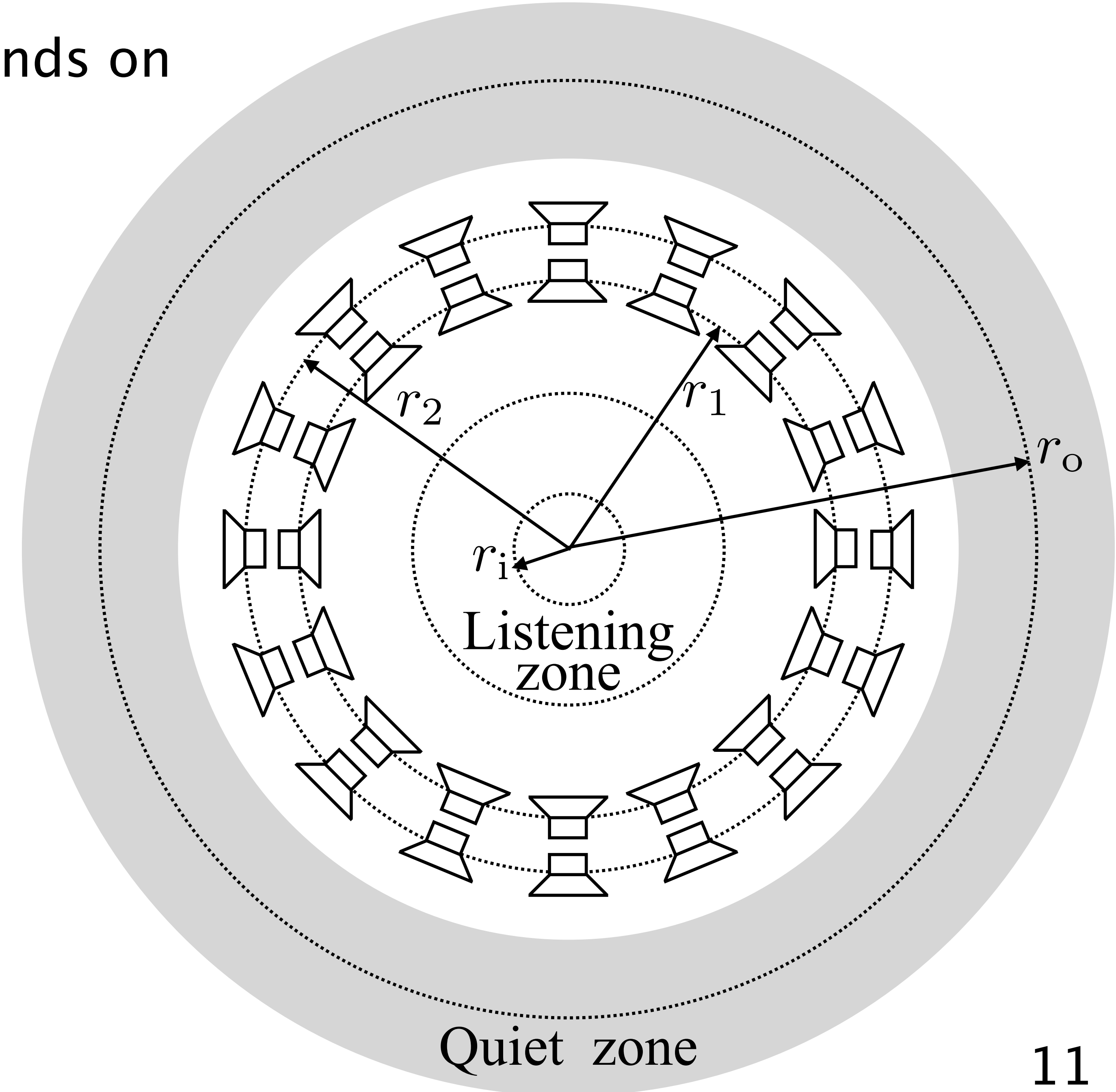
$$\mathring{D}_m = \frac{\mathring{S}_m(r_{\mathrm{ref}})}{2\pi \mathring{G}_{3\mathrm{D},m}(r_{\mathrm{ref}}, r_0)}$$

  - Proposed 2.5D HOA with a circular double-layer source
    - Two reference circles can be simultaneously controlled
  - Interior sound field

$$\mathring{S}_m(r_{\mathrm{i}}) = \mathring{A}_m J_m(kr_{\mathrm{i}})$$

  - Exterior sound field

$$\mathring{S}_m(r_{\mathrm{o}}) = \mathring{B}_m H_m(kr_{\mathrm{o}})$$



Proposed analytical driving functions

- Proposed interior and exterior sound field control
  - Interior control: desired sound pressures at inner reference circle

$$\mathring{S}_m(r_\mathrm{i}) = 2\pi \left\{ \mathring{D}_{m,1}\mathring{T}_{m,1}(r_{\mathrm{i}<}, r_1) + \mathring{D}_{m,2}\mathring{T}_{m,2}(r_{\mathrm{i}<}, r_2) \right\} = \mathring{A}_m J_m(kr_\mathrm{i})$$

  - Exterior control: no sound pressures at outer reference circle

$$\mathring{S}_m(r_\mathrm{o}) = 2\pi \left\{ \mathring{D}_{m,1}\mathring{T}_{m,1}(r_{\mathrm{o}>}, r_1) + \mathring{D}_{m,2}\mathring{T}_{m,2}(r_{\mathrm{o}>}, r_2) \right\} = \mathring{B}_m H_m(kr_\mathrm{o}) = 0$$

  - Matrix representation

$$2\pi \begin{bmatrix} \mathring{T}_{m,1}(r_{\mathrm{i}<}, r_1) & \mathring{T}_{m,2}(r_{\mathrm{i}<}, r_2) \\ \mathring{T}_{m,1}(r_{\mathrm{o}>}, r_1) & \mathring{T}_{m,2}(r_{\mathrm{o}>}, r_2) \end{bmatrix} \begin{bmatrix} \mathring{D}_{m,1} \\ \mathring{D}_{m,2} \end{bmatrix} = \begin{bmatrix} \mathring{A}_m J_m(kr_\mathrm{i}) \\ 0 \end{bmatrix}$$

  - Analytical driving functions

$$\mathring{D}_{m,1} = \frac{\mathring{T}_{m,2}(r_{\mathrm{o}>}, r_2)\mathring{A}_m J_m(kr_\mathrm{i})}{2\pi \mathring{T}_m(r_1, r_2, r_\mathrm{i}, r_\mathrm{o})} \qquad \mathring{D}_{m,2} = -\frac{\mathring{T}_{m,1}(r_{\mathrm{o}>}, r_1)\mathring{A}_m J_m(kr_\mathrm{i})}{2\pi \mathring{T}_m(r_1, r_2, r_\mathrm{i}, r_\mathrm{o})}$$

$$\mathring{T}_m(r_1, r_2, r_\mathrm{i}, r_\mathrm{o}) = \mathring{T}_{m,1}(r_{\mathrm{i}<}, r_1)\mathring{T}_{m,2}(r_{\mathrm{o}>}, r_2) - \mathring{T}_{m,2}(r_{\mathrm{i}<}, r_2)\mathring{T}_{m,1}(r_{\mathrm{o}>}, r_1)$$

Interior field control with $r_{\rm i} = 0$

- Interior field synthesis accuracy improvement
  - 2.5D HOA with interior reference radius $r = 0$ is best performance
  - Introducing L'Hôpital's rule (Zhang et al. IWAENC 2014.)

$$j_n(kr) \approx \frac{(kr)^n}{(2n+1)!!} \qquad J_m(kr) \approx \frac{\mathrm{sgn}(m)^{|m|}(kr)^{|m|}}{2^{|m|}|m|!} \qquad \text{I.e. } kr \to 0$$

  - Analytical Driving functions with interior reference radius $r_{\rm i} = 0$

$$\mathring{D}_{m,1}\Big|_{r_{\rm i}=0} = \frac{\mathring{T}_{m,2}(r_{\rm o>}, r_2)\mathring{A}_m C_m}{2\pi jkQ_{|m|}^m P_{|m|}^{|m|}(0)^2 U_m(a, r_1, r_2, r_{\rm o})}$$

$$\mathring{D}_{m,2}\Big|_{r_{\rm i}=0} = -\frac{\mathring{T}_{m,1}(r_{\rm o>}, r_1)\mathring{A}_m C_m}{2\pi jkQ_{|m|}^m P_{|m|}^{|m|}(0)^2 U_m(a, r_1, r_2, r_{\rm o})} \qquad C_m = \frac{\mathrm{sgn}(m)^{|m|}(2|m|+1)!!}{2^{|m|}|m|!}$$

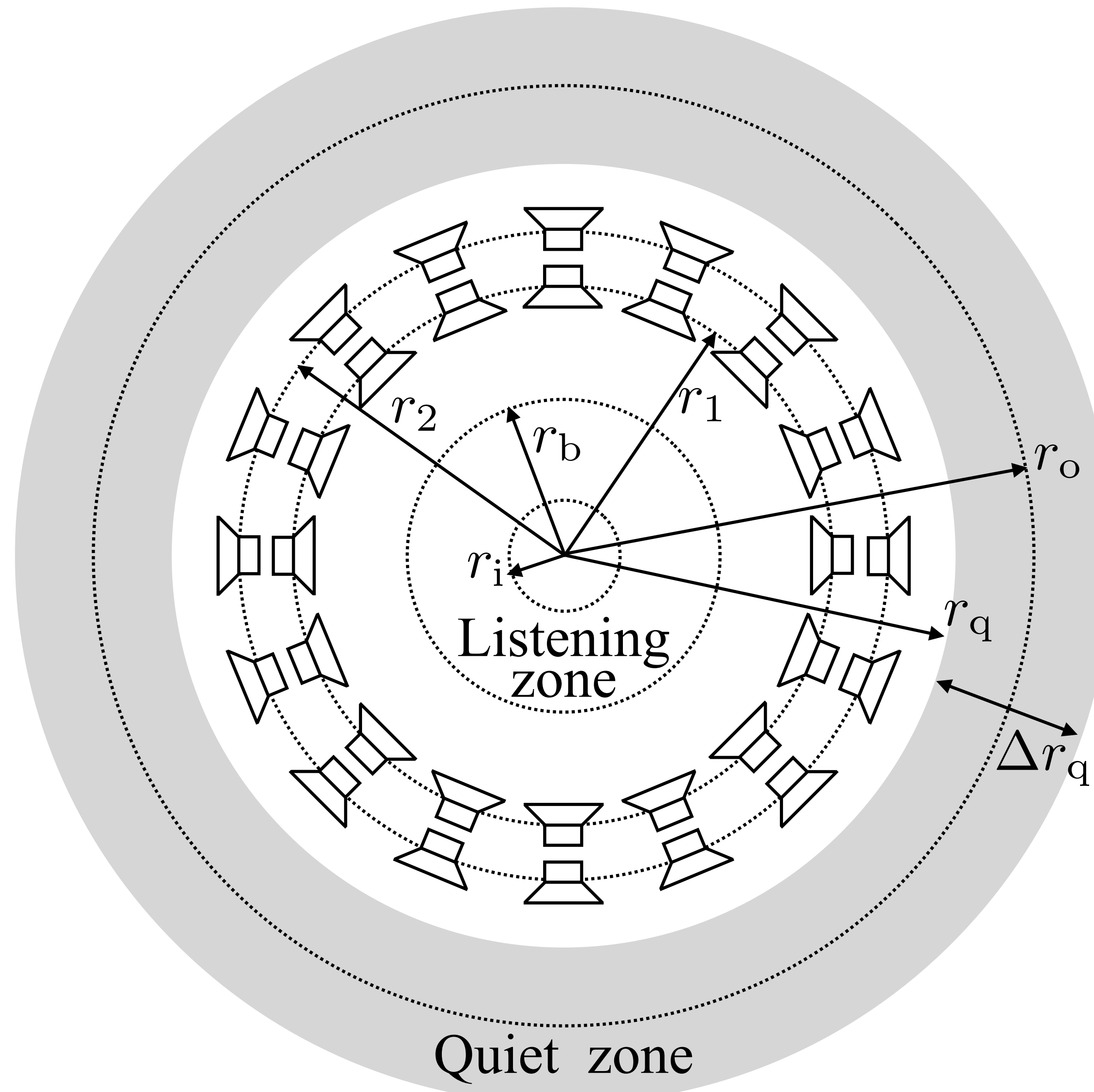
$$U_m(a, r_1, r_2, r_{\rm o}) = h_{|m|,1}(a, r_1)\mathring{T}_{m,2}(r_{\rm o>}, r_2) - h_{|m|,2}(a, r_2)\mathring{T}_{m,1}(r_{\rm o>}, r_1)$$

  - Driving signals with L loudspeakers on each layer

$$D(\phi_l) = \sum_{m=-M}^{M} \mathring{D}_m\Big|_{r_{\rm i}=0} e^{jm\phi_l} \qquad M = \lfloor (L-1)/2 \rfloor$$

Computer simulation setup

- Simulation condition
  - Speed of sound: 343.25 m/s
  - Inner and outer radii: 0.9 and 1.0 m
  - Inner control radius
    - LS: within 0.2 m
    - Proposed: 0 m
  - Outer control radius
    - LS: 2.0 to 3.0 m
    - Proposed: 2.5 m
  - Number of loudspeakers / layer: 32
  - Maximum Fourier series order: 15
  - Control grid distance in LS: 0.05 m
  - Target sound field: Plane wave

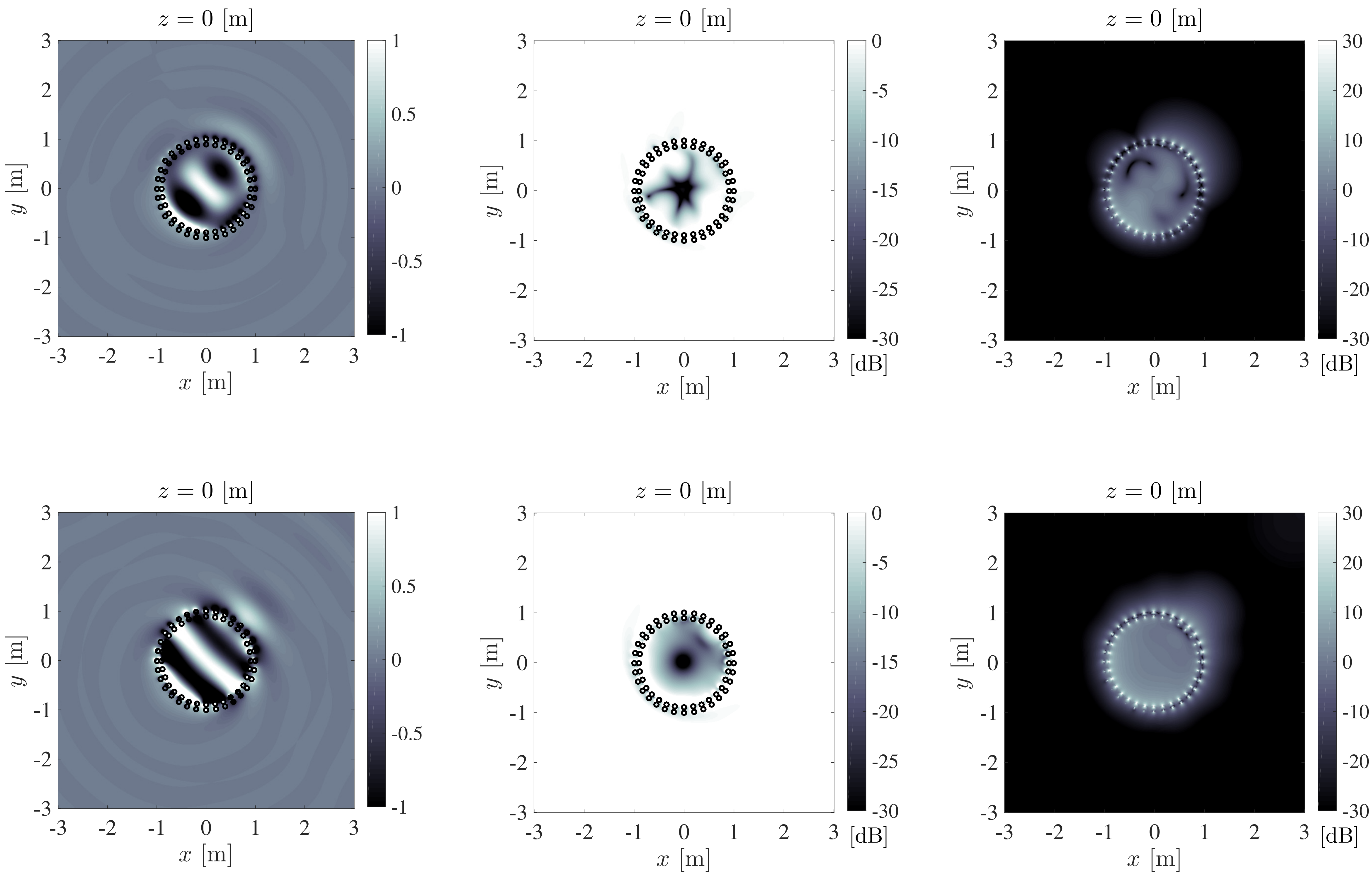


- Evaluation values
  - Synthesis error
  - Acoustic contrast between quiet zone and synthesis center

Results ($f = 400$ Hz, cardioid)

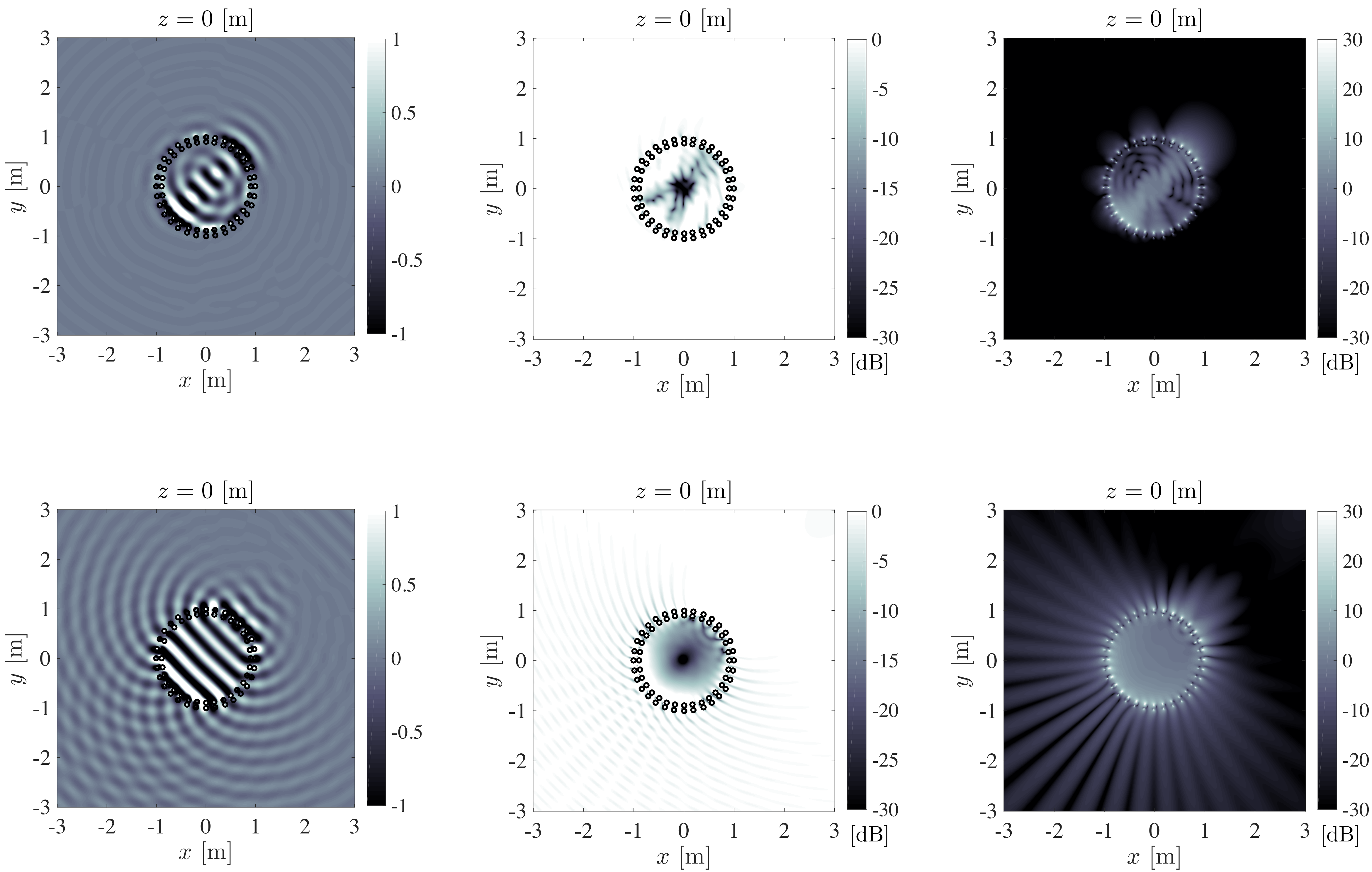
Conventional LS

Proposed



Synthesized field Synthesis error Acoustic contrast

Results ($f =$ 1k Hz, cardioid)



Conventional LS

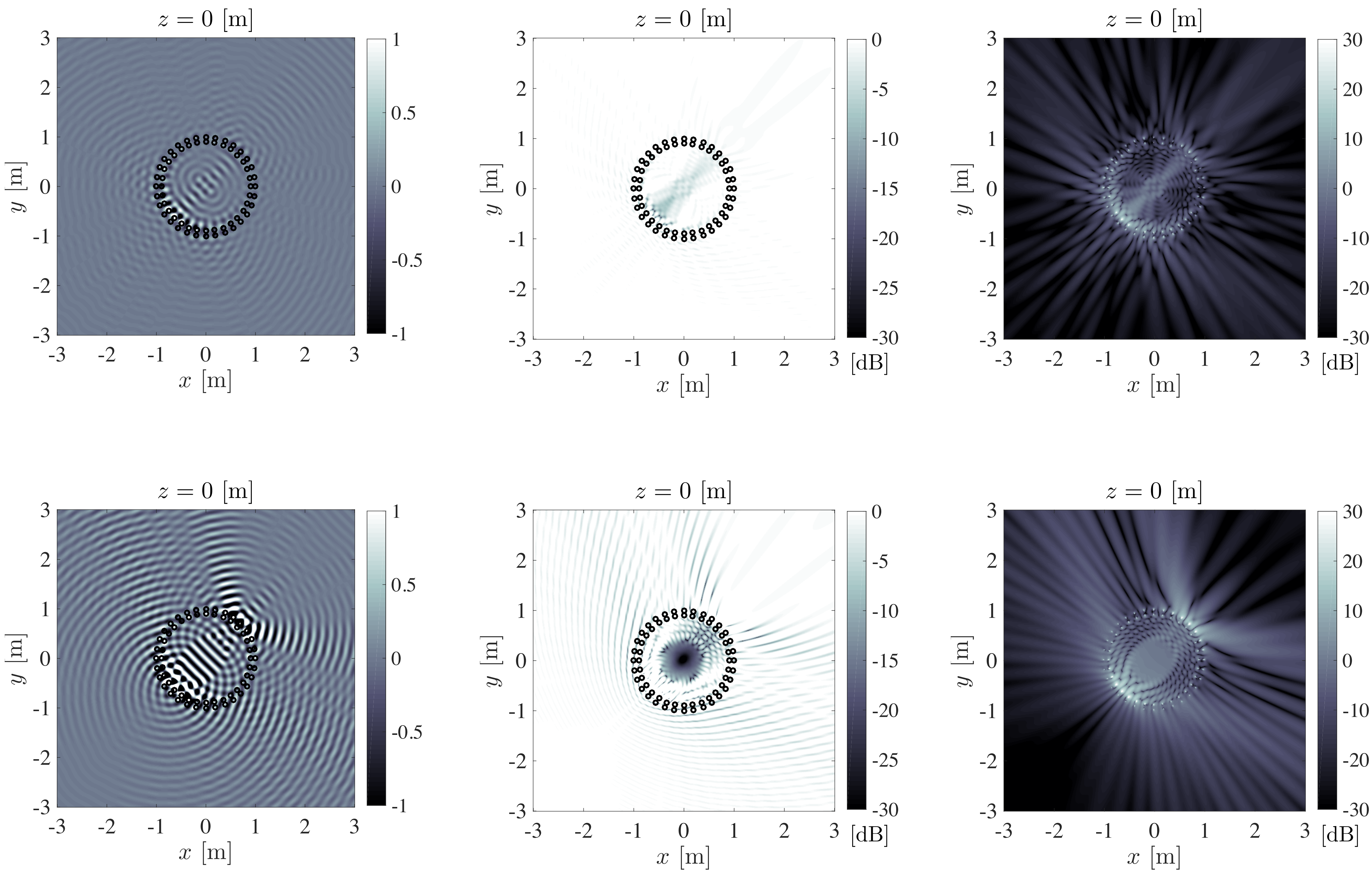
Proposed

Synthesized field | Synthesis error | Acoustic contrast

Results (f = 2k Hz, cardioid)

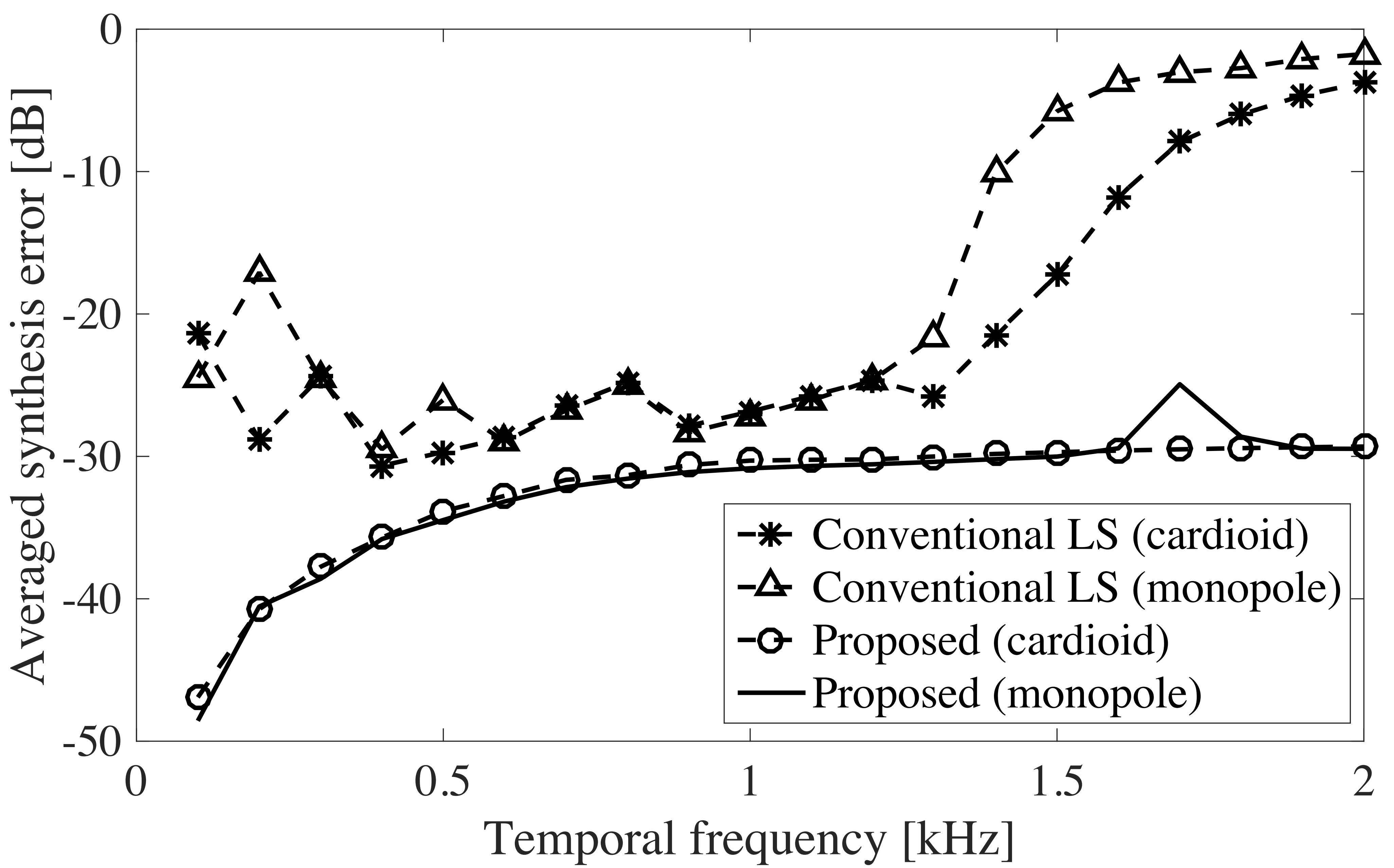
Conventional LS

Proposed



Synthesized field | Synthesis error | Acoustic contrast

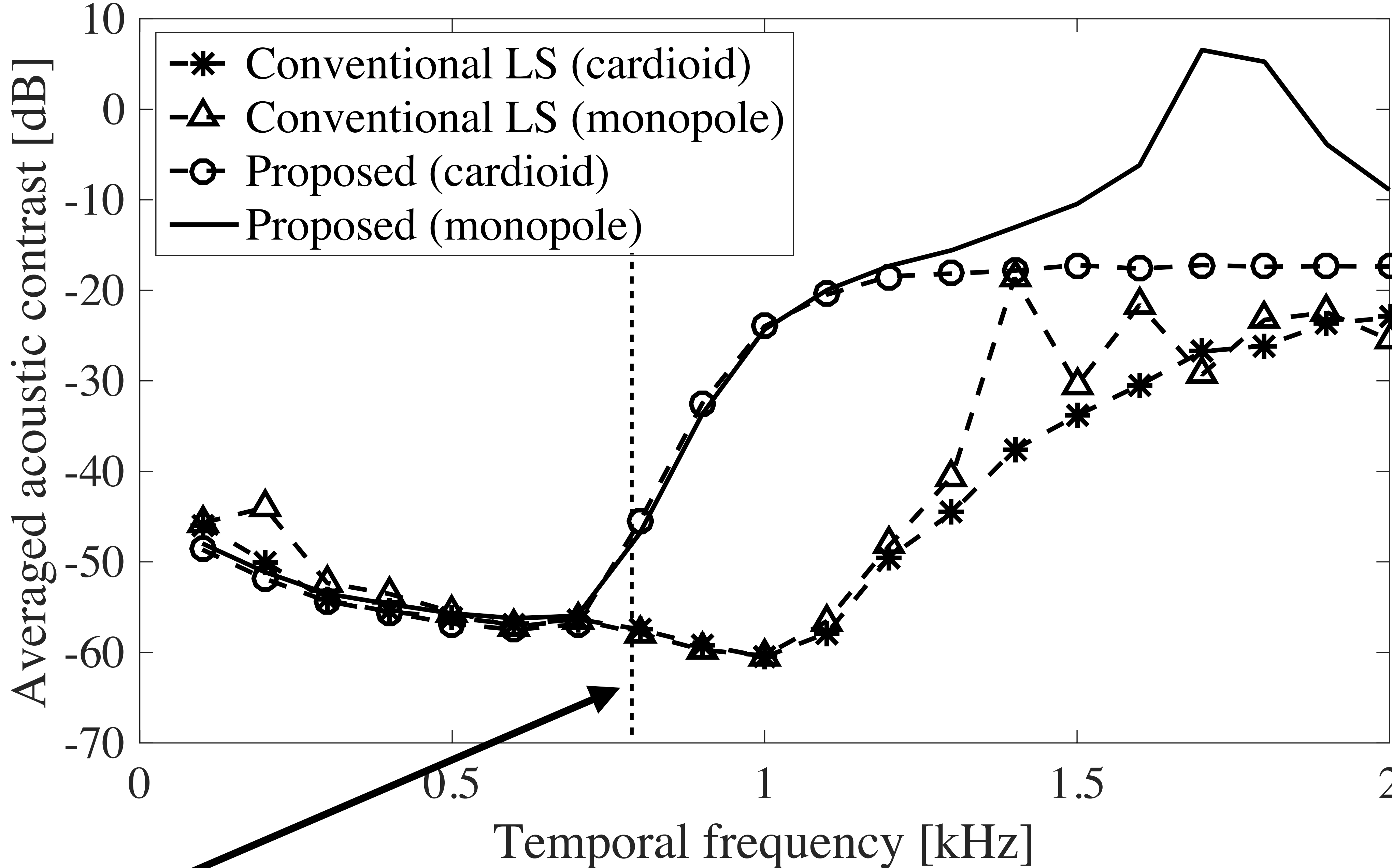
Results of averaged synthesis accuracy



$$E(\boldsymbol{r}) = 10 \log_{10} \frac{|S_{\mathrm{des}}(\boldsymbol{r}) - S_{\mathrm{syn}}(\boldsymbol{r})|^2}{|S_{\mathrm{des}}(\boldsymbol{r})|^2}$$

Proposal outperforms LS method at all temporal frequencies

Results of averaged acoustic contrast



Spatial Nyquist frequency
of outer array $f = 819$ Hz

$$C(\boldsymbol{r}) = 10 \log_{10} \frac{|S_{\mathrm{syn}}(\boldsymbol{r})|^2}{|S_{\mathrm{syn}}(r=0)|^2}$$

Concluding remarks

- Analytical approach to 2.5D sound field control with a circular double-layer array of fixed directivity loudspeakers
  - Regularization free approach
  - Analytical driving functions of fixed directivity loudspeakers
  - Evaluation by computer simulations

- Acknowledgement
  - This study was partly supported by JSPS KAKENHI Grant Number 15K21674.