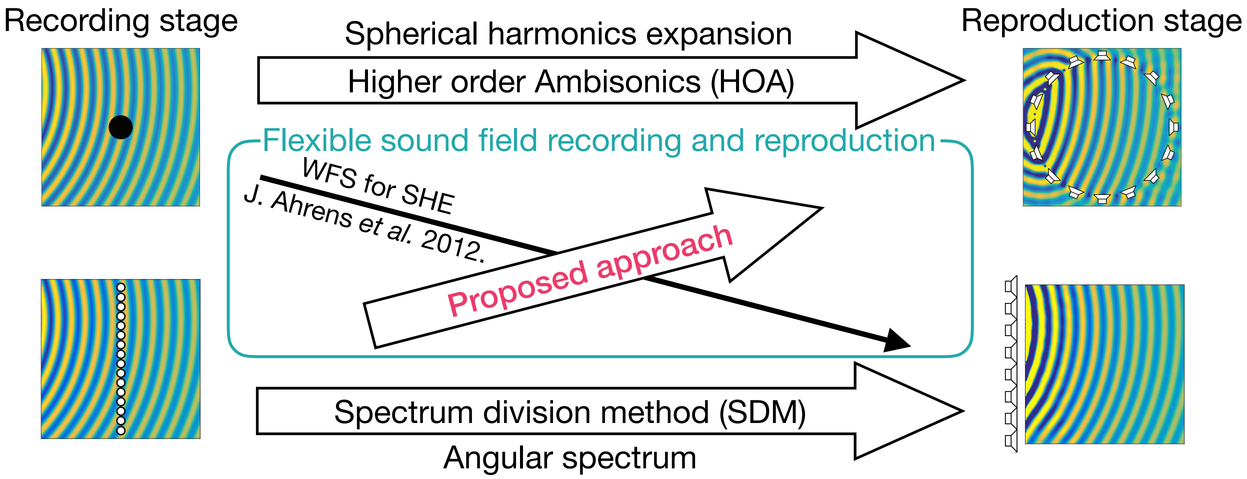
2.5D higher-order Ambisonics for a sound field described by angular spectrum coefficients

Takuma OKAMOTO, National Institute of Information and Communication Technology, Japan

1. Introduction

- **Analytical approaches to sound field recording and reproduction**

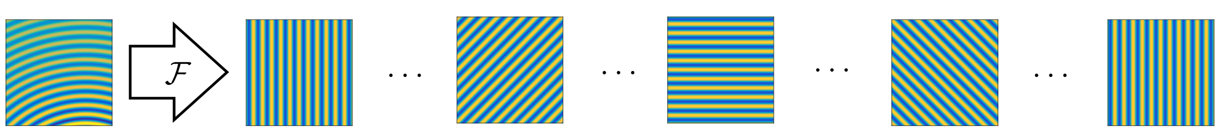


- **Analytical conversion of sound field representation**
  - Analytical driving function of 2.5D synthesis for circular secondary sources
    - Sound field recorded by linear microphone arrays can be reproduced by circular loudspeaker arrays

2. Analytical conversion of sound field representation

- **Sound field represented by angular spectrum coefficients**

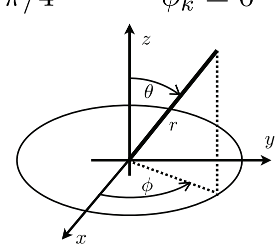
$$P(x,y,\omega) \simeq \frac{1}{2\pi}\int_{-k}^{k} \tilde{P}(k_x,\omega) e^{-j\sqrt{k^2-k_x^2}y} e^{-jk_x x} dk_x$$ Spatial inverse Fourier transform



$\tilde{P}(k_x=-k)$, $\phi_k=\pi$ … $\tilde{P}(k_x=-k/2)$, $\phi_k=3\pi/4$ … $\tilde{P}(k_x=0)$, $\phi_k=\pi/2$ … $\tilde{P}(k_x=k/2)$, $\phi_k=\pi/4$ … $\tilde{P}(k_x=k)$, $\phi_k=0$

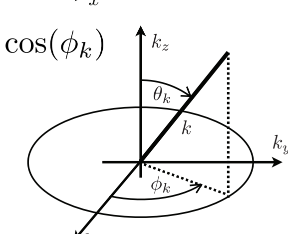
- **Circular harmonics expansion of plane wave**

$$e^{-jkr\cos(\phi-\phi_k)} = \sum_{m=-\infty}^{\infty} (-j)^m J_m(kr) e^{jm(\phi-\phi_k)}$$



- **Analytical conversion of sound field representation from angular spectrum to circular harmonics expansion**
  - Coordinate system is transformed as $[x,\ y] \to [r,\ \phi],\ k_x \to k\cos(\phi_k)$

$$P(r,\phi,\omega) = \frac{k}{2\pi}\int_0^{\pi} \tilde{P}(\phi_k,\omega) e^{-jkr\cos(\phi-\phi_k)} \sin(\phi_k) d\phi_k$$



  - Analytical solution for 2D sound field

$$\mathring{P}_{2\mathrm{D},m}(r,\omega) = \frac{k(-j)^m J_m(kr)}{2\pi}\int_0^{\pi} \tilde{P}(\phi_k,\omega)\sin(\phi_k) e^{-jm\phi_k} d\phi_k$$

3. Proposed formulation

- **Sound field recorded by linear receiver** (Spatial Fourier transform)

$$\tilde{P}(\phi_k,\omega) = \tilde{P}(k_x,\omega) = \int_{-\infty}^{\infty} P(x,0,\omega) e^{jk_x x} dx = \int_{-\infty}^{\infty} P(x,0,\omega) e^{jk\cos(\phi_k)x} dx$$

- **2.5D HOA for circular secondary sources** (J. Ahrens *et al.* 2008.)

$$\mathring{D}_{2.5\mathrm{D},m}(r=0,r_0,\omega) = \frac{1}{2\pi r_0}\cdot\frac{\breve{P}^m_{|m|}(\omega)}{-jkh^{(2)}_{|m|}(kr_0) Y^m_{|m|}(\pi/2,0)^*}$$

Plane wave : $\breve{P}^m_{|m|}(\phi_k,\omega) = 4\pi(-j)^{|m|} Y^m_{|m|}(\pi/2,\phi_k)^*$

Point source : $\breve{P}^m_{|m|}(\boldsymbol{r}_s,\omega) = -jkh^{(2)}_{|m|}(kr_s) Y^m_{|m|}(\theta_s,\phi_s)^*$

Integrated with $(k/2\pi)\tilde{P}(\phi_k,\omega)\sin(\phi_k)$ from $\phi_k=0$ to π

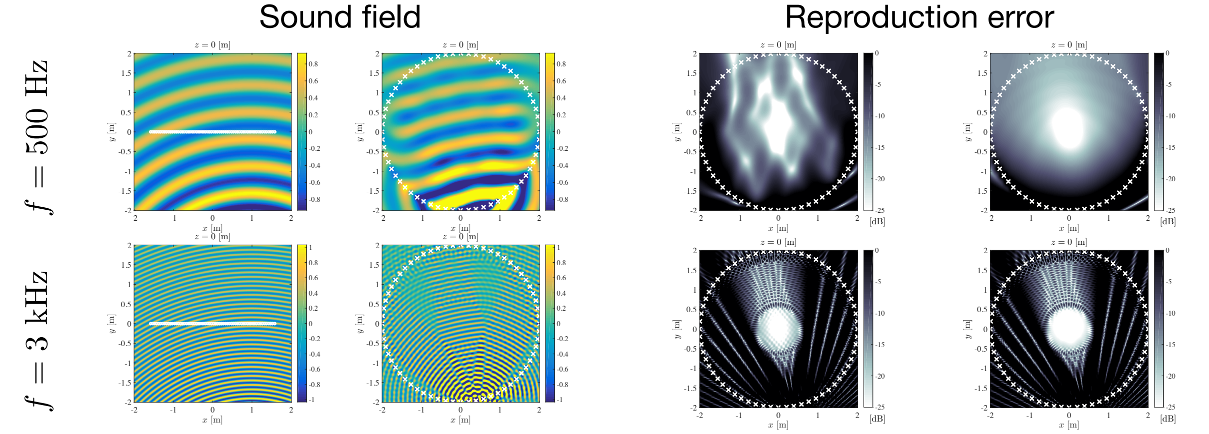
- **Analytical driving function of 2.5D synthesis**

$$\mathring{D}_{2.5\mathrm{D},m}(r_0,\omega) = \frac{(-j)^{|m|-1}}{\pi r_0 h^{(2)}_{|m|}(kr_0)}\int_0^{\pi} \tilde{P}(\phi_k,\omega)\sin(\phi_k) e^{-jm\phi_k} d\phi_k$$

4. Computer simulations

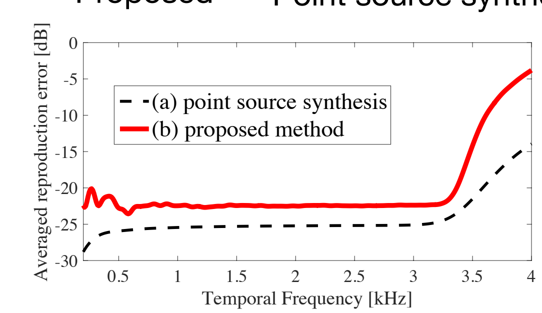
- **Reproduced sound field and reproduction error**

$L_{\mathrm{mic}} = L_{\mathrm{sp}} = 64$ ch, $\Delta x = 0.05$ m, $r_0 = 2.0$ m, $\boldsymbol{x}_s = [1, -5,\ 0]^{\mathrm{T}}$



- **Averaged reproduction error** ($r \le 0.5$ m)

$$E(\boldsymbol{r},\omega) = 10\log_{10}\frac{|P_{\mathrm{org}}(\boldsymbol{r},\omega) - P_{\mathrm{syn}}(\boldsymbol{r},\omega)|^2}{|P_{\mathrm{org}}(\boldsymbol{r},\omega)|^2}$$



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