3D LOCALIZED SOUND ZONE GENERATION WITH A PLANAR OMNI-DIRECTIONAL LOUDSPEAKER ARRAY

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ABSTRACT

This paper provides a 3D localized sound zone generation method using a planar omni-directional loudspeaker array. In the proposed method, multiple co-centered circular arrays are arranged on the horizontal plane and an additional loudspeaker is located at the array's center. The sound field produced by this center loudspeaker is then cancelled using the multiple circular arrays. A localized 3D sound zone can thus be generated inside a sphere with a maximum radius of that of the circular arrays because the residual sound field is contained within the sphere. The resulting sound fields are decomposed into spherical harmonic spectra and the driving function of the array is then obtained. Compared with the conventional approach that uses monopole pairs to control the even and odd spherical harmonic spectrum components, the proposed method can be simply realized with a practical planar omni-directional array because it is sufficient to control the 0-th order component. Computer simulations confirm the effectiveness of the proposed approach.

Index Terms— Sound field synthesis, localized sound zone, circular loudspeaker array, spherical harmonic expansion, Ambisonics.

1. INTRODUCTION

Generation of a personalized sound zone using loudspeakers is an attractive and important acoustic application. Many methods to realize personalized sound zones have been investigated and can be broadly categorized into two main schemes. The first scheme controls either the acoustic contrast or the energy between two spaces [1–8]. In contrast, the second approach controls multiple sound fields [9–18] such that they are simultaneously synthesized for different zones in space. These approaches typically control sound zones located away from the loudspeakers.

In an alternative approach, a localized sound zone is generated near loudspeakers such that the audible sound pressure only propagates close to the loudspeakers, while it propagates at very low amplitudes beyond a reference distance. This approach is called localized sound zone generation [19–21]. This technique is better suited to actual implementation than the other methods mentioned above, which control sound zones located away from the loudspeakers, because the sound pressures are not radiated at the walls and no sound reflections occur. Several localized sound zone generation methods have been proposed.

One such method is based on evanescent wave reproduction [22] using linear or circular loudspeaker arrays [23, 24]. In these methods, however, the distance attenuation properties at high frequencies are too short, and the listening area becomes too small because the evanescent waves decay in less than one wavelength [22].

Additionally, analytical approaches using linear, circular or spherical arrays along with an additional loudspeaker have also been proposed [19–21]. In these methods, the sound field produced by the additional loudspeaker is cancelled by the sound fields of the linear, circular or spherical arrays and a localized sound zone can thus be generated as a residual sound field. However, undesired sound pressures propagate toward the vertical angles in the methods using the linear or circular arrays [19, 21] because they are based on 2.5D sound field synthesis [25, 26] and only sound pressures on the horizontal plane can be controlled. While the method using the spherical array can realize 3D localized sound zone generation, this method is impractical in terms of both array configuration and listening area because large numbers of loudspeakers are required to form the surrounding spherical array and the listening area is located inside this spherical array [20].

To address the aforementioned problems, this paper provides a 3D localized sound zone generation method with a practical planar omni-directional loudspeaker array. The proposed method is based on 3D exterior sound field synthesis using a planar array [27, 28].

In [28], undesired sound pressures caused by acoustic noise sources are cancelled using multiple co-centered circular arrays located on the horizontal plane. In this case, only the sound pressure outside a sphere with a maximum radius of that of the circular arrays can be cancelled in principle; however, the residual sound pressures remain inside the sphere as in [28]. Using this property, an additional loudspeaker is introduced in place of noise sources in [28] and the sound field produced by the loudspeaker is then cancelled using the circular arrays in the proposed approach, as in [19–21]. 3D localized sound zone generation can thus be realized inside a sphere with a maximum radius of that of the circular arrays.

In [27, 28], however, both omni-directional and first-order loudspeakers are required to control $\breve{B}_n^m$ for $n + |m|$ even and odd spherical harmonic spectrum components of the 3D exterior sound field. For actual implementation, first-order loudspeakers are approximately realized using monopole pairs [27, 28] as 3D sound field recording with a planar microphone array [29]. In the approach proposed here, the additional loudspeaker is located on the horizontal plane and the sound field produced by this loudspeaker contains only even components, as noted in [28, 30]. This means that it is sufficient to use omni-directional circular arrays. Additionally, by locating the additional loudspeaker at the array's center, the sound field produced by the loudspeaker only contains the 0-th order component and the driving function for the circular arrays can then be

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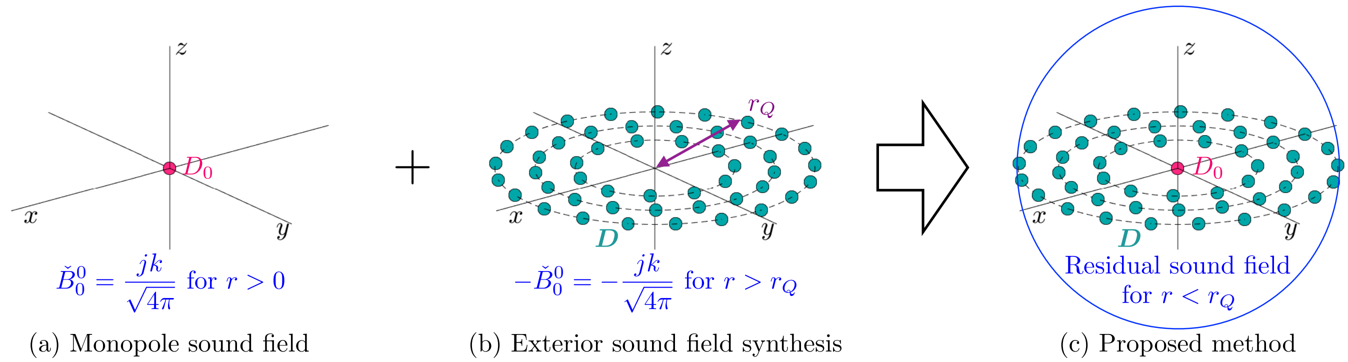


Figure 1: Scheme for proposed 3D localized sound zone generation method using planar omni-directional loudspeaker array: (a) sound field produced by monopole loudspeaker located at origin with driving function D_0 that only contains 0-th order spherical harmonic spectrum component $\check{B}_0^0$ for $r > 0$; (b) $-\check{B}_0^0$ for $r > r_Q$ can be synthesized using multiple circular arrays with maximum radius r_Q and driving function $\boldsymbol{D}$; (c) 3D localized sound zone that can be generated as a residual sound field for $r < r_Q$ by simultaneously driving D_0 and $\boldsymbol{D}$.

simplified. Compared with the existing approaches using monopole pairs [27, 28], the proposed method can be simply realized using a practical planar omni-directional array with half the typical number of loudspeakers. Additionally, the array configuration and listening area in the proposed method are more practical than those of the conventional evanescent and residual approaches [19–21, 23, 24]. The scheme for the proposed method is shown in Fig. 1.

2. 3D EXTERIOR SOUND FIELD PRODUCED BY POINT SOURCE ON HORIZONTAL PLANE

In this paper, the spherical coordinates relative to Cartesian coordinates are defined as used in [22].

The exterior expansion of the 3D sound field for regions exterior to any sound sources is given as:

$$S(r,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \check{B}_n^m h_n(kr) Y_n^m(\theta,\phi), \tag{1}$$

where $\check{B}_n^m$ and h_n are the spherical harmonic spectra of the exterior sound field and the n-th order spherical Hankel function of the first kind, respectively, and k is the wavenumber [22];

$$Y_n^m(\theta,\phi) = \underbrace{\sqrt{\frac{(2n+1)}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}} P_n^{|m|}(\cos\theta)}_{\mathcal{P}_n^{|m|}(\cos\theta)} e^{jm\phi} \tag{2}$$

represents the spherical harmonics; $P_n^{|m|}$ is the associated Legendre function [31]. Under the free-field assumption, the transfer function of a point source located at $\boldsymbol{r}_0 = [r, \theta_0, \phi_0]^{\mathrm{T}}$ to a point $\boldsymbol{r} = [r, \theta, \phi]^{\mathrm{T}}$ is given by the 3D free-field Green's function [22]:

$$\begin{aligned} G(\boldsymbol{r},\boldsymbol{r}_0) &= \frac{e^{jk|\boldsymbol{r}-\boldsymbol{r}_0|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}_0|} \\ &= jk\sum_{n=0}^{\infty}\sum_{m=-n}^{n} j_n(kr_0)h_n(kr)Y_n^m(\theta_0,\phi_0)^* Y_n^m(\theta,\phi), \end{aligned} \tag{3}$$

for $r > r_0$, where $j = \sqrt{-1}$ and j_n is the n-th order spherical Bessel function [22]. When the point source is located on the horizontal plane with $\theta_0 = \pi/2$, (3) is represented by:

$$\begin{aligned} &G(\boldsymbol{r},\boldsymbol{r}_0)|_{\theta_0=\pi/2} \\ &= jk\sum_{n=0}^{\infty}\sum_{m=-n}^{n} j_n(kr_0)h_n(kr)\mathcal{P}_n^{|m|}(0)e^{-jm\phi_0}Y_n^m(\theta,\phi). \end{aligned} \tag{4}$$

From (4), the sound field produced by the point source located on the horizontal plane only includes the spherical harmonic spectra for $n+|m|$ even because $\mathcal{P}_n^{|m|}(0) = 0$ when $n+|m|$ is odd [27–30]. Therefore, to synthesize a complete 3D exterior sound field using loudspeakers on the horizontal plane, both monopole and vertical dipole components are required. These vertical dipole components include the vertical derivatives of $\mathcal{P}_n^{|m|}(0)$ and can control the odd spectra. To approximately implement the vertical dipole components using actual monopole loudspeakers, monopole pairs are introduced [27, 28]. Therefore, the conventional approach requires double the number of loudspeakers.

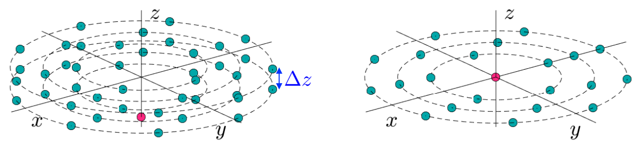
In the proposed approach, the sound field produced by a point source located at the origin only including the 0-th order spherical harmonic spectrum component (Fig. 1(a)), is cancelled using multiple co-centered circular loudspeaker arrays on the horizontal plane (Fig. 1(b)). As a result, the proposed method can be simply realized using a practical planar omni-directional array with half the number of loudspeakers required for the conventional method using monopole pairs [27, 28].

3. PROPOSED 3D LOCALIZED SOUND ZONE GENERATION

From (3), the sound field produced by a monopole loudspeaker with driving function D_0 located at the origin (Fig. 1(a)) is given as:

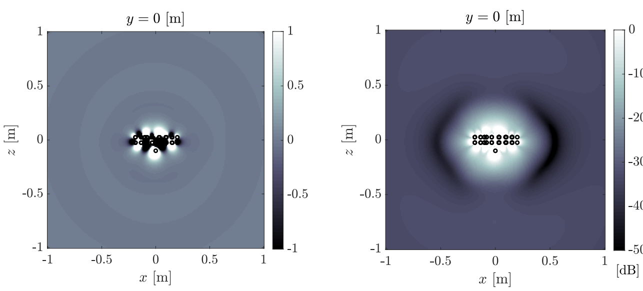
$$\begin{aligned} S_0(r,\theta,\phi) &= D_0 G(\boldsymbol{r}, r_0 = 0) \\ &= D_0\sum_{n=0}^{0}\sum_{m=-n}^{n}\frac{jk}{\sqrt{4\pi}}h_n(kr)Y_n^m(\theta,\phi) = D_0\frac{e^{jkr}}{4\pi r}, \end{aligned} \tag{5}$$

where $j_0(0) = 1$, $j_n(0) = 0$ for $n > 0$, $\mathcal{P}_0^0(0) = 1/\sqrt{4\pi}$ and $h_0(kr) = e^{jkr}/jkr$ [22]. Subsequently, D_0 is set at 1 for simplicity. From (1), S_0 only has the $B_0^0 = jk/\sqrt{4\pi}$ component for



(a) Method with monopole pairs (b) Proposed method

Figure 2: Arrangements of loudspeakers with three radii $r_{1,2,3} = 0.1$, 0.15 and 0.2 m, where these circular arrays contain five, seven and nine loudspeakers, respectively: (a) conventional method using monopole pairs ($\Delta z = 0.05$ m) with 43 loudspeakers, where the loudspeaker for D_0 is located at $[0,\ 0,\ \ -0.1]^{\mathrm{T}}$ in Cartesian coordinates; (b) proposed method with planar array of 22 loudspeakers.



(a) $S_{0+1}(x, y = 0, z)$ (b) $20 \log_{10} |S_{0+1}(x, y = 0, z)|$

Figure 3: Results of conventional method using monopole pairs for $f = 500$ Hz: (a) residual sound field $S_{0+1}(x, y = 0, z)$ produced by D_0 and $\boldsymbol{D}^{\text{even+odd}}$; (b) sound pressure level of residual sound field $20 \log_{10} |S_{0+1}(x, y = 0, z)|$.

$r > 0$. In the proposed approach, S_0 is cancelled using multiple co-centered circular arrays.

A sound field synthesized using a continuous circular sound source with radius r_q on the horizontal plane is given as:

$$S_1(r, \theta, \phi) = \int_0^{2\pi} D(r_q, \phi_0)\ G(\boldsymbol{r}, \boldsymbol{r}_q)|_{\theta_0 = \pi/2}\, d\phi_0, \tag{6}$$

where $D(r_q, \phi_0)$ is the circular source driving function and $\boldsymbol{r}_q = [r_q, \theta_0 = \pi/2, \phi_0]^{\mathrm{T}}$. As in [27, 28], $D(r_q, \phi_0)$ is decomposed into the 2D cylindrical harmonic spectra as follows:

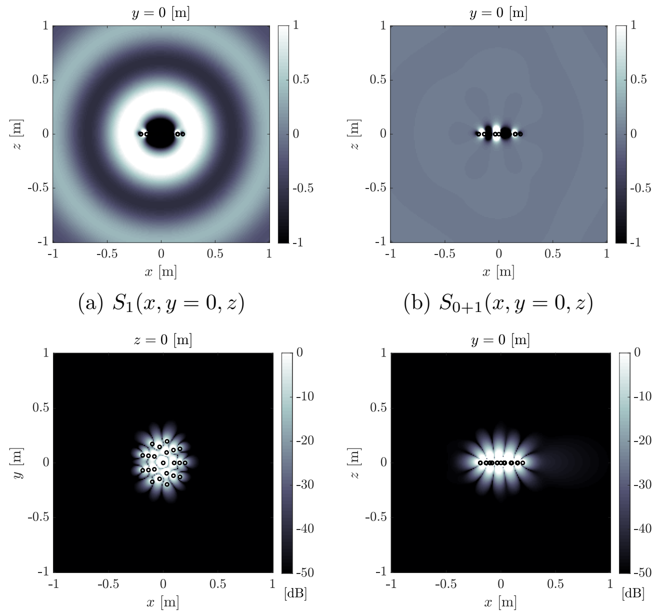
$$D(r_q, \phi_0) = \sum_{m=-\infty}^{\infty} \mathring{D}_{m,q} e^{jm\phi_0}. \tag{7}$$

From (4) and (7), and by introducing multiple radii (i.e., $q = 1, 2, \cdots, Q$), (6) can be represented as:

$$\begin{aligned} S_1(r, \theta, \phi) &= \sum_{q=1}^{Q} \int_0^{2\pi} D(r_q, \phi_0)\ G(\boldsymbol{r}, \boldsymbol{r}_q)|_{\theta_0 = \pi/2}\, d\phi_0 \\ &= 2\pi jk \sum_{q=1}^{Q} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \mathring{D}_{m,q} \mathcal{P}_n^{|m|}(0) j_n(kr_q) h_n(kr) Y_n^m(\theta, \phi). \end{aligned} \tag{8}$$

In [28], the primary sound field B_n^m is cancelled using the multiple circular arrays. In this case, the relationship

$$\breve{B}_n^m + 2\pi jk \sum_{q=1}^{Q} \mathring{D}_{m,q} j_n(kr_q) \mathcal{P}_n^{|m|}(0) = 0 \tag{9}$$



(a) $S_1(x, y = 0, z)$ (b) $S_{0+1}(x, y = 0, z)$

(c) $20 \log_{10} |S_{0+1}(x, y, z = 0)|$ (d) $20 \log_{10} |S_{0+1}(x, y = 0, z)|$

Figure 4: Results of proposed method for $f = 500$ Hz: (a) sound field $S_1(x, y = 0, z)$ produced by $\boldsymbol{D}$ using 21 loudspeakers; (b) residual sound field $S_{0+1}(x, y = 0, z)$ produced using D_0 and $\boldsymbol{D}$; (c), (d) sound pressure levels for the residual sound fields $20 \log_{10} |S_{0+1}(x, y, z = 0)|$ and $20 \log_{10} |S_{0+1}(x, y = 0, z)|$, respectively.

is obtained from (1) and (8). When the maximum order in (8) is N, (9) can be represented in matrix form as:

$$\frac{1}{2\pi jk} \mathring{\boldsymbol{B}}_m^{\text{even}} \simeq \boldsymbol{U}_{|m|} \check{\boldsymbol{D}}_m^{\text{even}}, \tag{10}$$

where

$$\breve{\boldsymbol{B}}_m^{\text{even}} = -\left[\breve{B}_{|m|}^m,\ \breve{B}_{|m|+2}^m,\ \cdots,\ \breve{B}_N^m\right]^{\mathrm{T}}, \tag{11}$$

$$\boldsymbol{U}_{|m|} = \begin{bmatrix} U_{|m|}^{|m|}(r_1) & U_{|m|}^{|m|}(r_2) & \cdots & U_{|m|}^{|m|}(r_Q) \\ U_{|m|+2}^{|m|}(r_1) & U_{|m|+2}^{|m|}(r_2) & \cdots & U_{|m|+2}^{|m|}(r_Q) \\ \vdots & \vdots & \vdots & \vdots \\ U_N^{|m|}(r_1) & U_N^{|m|}(r_2) & \cdots & U_N^{|m|}(r_Q) \end{bmatrix}, \tag{12}$$

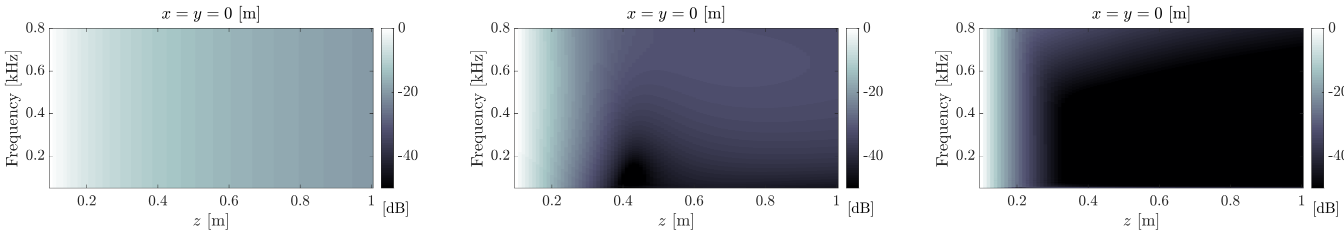
$$U_n^{|m|}(r_q) = j_n(kr_q) \mathcal{P}_n^{|m|}(0), \tag{13}$$

and

$$\mathring{\boldsymbol{D}}_m^{\text{even}} = \left[\mathring{D}_{m,1},\ \mathring{D}_{m,2},\ \cdots,\ \mathring{D}_{m,Q}\right]^{\mathrm{T}}, \tag{14}$$

in the case where both N and m are either odd or even; otherwise, replace N in (11) and (12) with $N - 1$. Then, the driving function of the circular arrays to synthesize the sound field $-B_n^m$ for $n + |m|$ even can be obtained as [28]:

$$\mathring{\boldsymbol{D}}_m^{\text{even}} = \frac{1}{2\pi jk} \left(\boldsymbol{U}_{|m|}^{\mathrm{T}} \boldsymbol{U}_{|m|}\right)^{-1} \boldsymbol{U}_{|m|}^{\mathrm{T}} \breve{\boldsymbol{B}}_m^{\text{even}}. \tag{15}$$



(a) Point source (b) Method with monopole pairs (c) Proposed method

Figure 5: Sound pressure level distributions for simple point source, the conventional method using monopole pairs, and the proposed method.

In the proposed approach, the synthesized sound field is not (11) but (5). B_n^m thus only contains the 0-th order component as:

$$\breve{\boldsymbol{B}}_0^{\text{even}} = -\left[\frac{jk}{\sqrt{4\pi}},\ 0,\ \cdots,\ 0\right]^{\mathrm{T}}. \tag{16}$$

Then, the driving function also only contains the 0-th order 2D cylindrical harmonic component and can be simply obtained as:

$$\mathring{\boldsymbol{D}}_0 = \boldsymbol{D} = -\frac{1}{8\pi^2}\left(\boldsymbol{U}_0^{\mathrm{T}}\boldsymbol{U}_0\right)^{-1}\boldsymbol{J}_0, \tag{17}$$

where

$$\boldsymbol{D} = \left[D(r_1, \phi_0),\ D(r_2, \phi_0),\ \cdots,\ D(r_Q, \phi_0)\right]^{\mathrm{T}}, \tag{18}$$

$$\boldsymbol{U}_0^{\mathrm{T}}\breve{\boldsymbol{B}}_0^{\text{even}} = -\frac{jk}{\sqrt{4\pi}}\frac{1}{\sqrt{4\pi}}\boldsymbol{J}_0 = -\frac{jk}{4\pi}\boldsymbol{J}_0, \tag{19}$$

$$\boldsymbol{J}_0 = \left[j_0(kr_1),\ j_0(kr_2),\ \cdots,\ j_0(kr_Q)\right]^{\mathrm{T}}, \tag{20}$$

and $\boldsymbol{D}$ is obviously independent of ϕ_0 and can be represented using real number coefficients. The proposed approach is thus a special case of the conventional method [28] when only cancelling $B_0^0 = jk/\sqrt{4\pi}$. Compared with S_0, however, the $-B_0^0$ produced by $\boldsymbol{D}$ in (17) only holds for $r > r_Q$, where r_Q is the maximum circular array radius because the sound sources are located in $r \leq r_Q$. Consequently, by simultaneously driving D_0 and $\boldsymbol{D}$, a 3D localized sound zone can be generated inside a sphere with r_Q as the residual sound field between S_0, given by (5), and S_1, given by (17) (Fig. 1(c)). In the proposed method, the listening area is entirely inside the sphere, except for the horizontal plane on which the loudspeakers are arranged.

4. COMPUTER SIMULATIONS

Computer simulations were performed to evaluate the proposed approach and compare it with the conventional method using monopole pairs [28]. In all simulations, a 3D free-field was assumed and the speed of sound c was 343.36 m/s. Three co-centered omni-directional circular loudspeaker arrays were introduced on the horizontal plane and their radii were set to 0.1, 0.15, and 0.2 m. The target frequency band ranged up to 800 Hz, as in [28]. The numbers of loudspeakers on these circular arrays were calculated to be five, seven and nine, respectively, using an array design procedure given in [28,29]. The total numbers of loudspeakers for the conventional and proposed methods were thus 43 and 22, respectively. In the conventional method using monopole pairs, both the $\boldsymbol{B}_m^{\text{even}}$ and $\boldsymbol{B}_m^{\text{odd}}$ components were controlled, as in [28]. In the simulations of the conventional method, the loudspeaker for D_0 was located at $[0, 0, -0.1]^{\mathrm{T}}$ in Cartesian coordinates. In this case, the sound field produced by D_0 contains only $\boldsymbol{B}_0^{\text{even}}$ and $\boldsymbol{B}_0^{\text{odd}}$ components and the driving function for the conventional method, $\boldsymbol{D}^{\text{even+odd}}$, was also independent of ϕ_0. The maximum order N in (12) was set at 100. The loudspeaker arrangements for both the conventional and proposed methods are depicted in Fig. 2.

The results for sound fields $S_1(x, y, z)$ and $S_{0+1}(x, y, z)$, and their sound pressure levels, $20\log_{10}|S_{0+1}(x, y, z)|$, produced using the conventional approach with monopole pairs and using the proposed method with a planar omni-directional array at a frequency of $f = 500$ Hz are shown in Figs. 3 and 4, respectively. Additionally, the results for the sound pressure levels on the z-axis (0.1 m $\leq z \leq 1.0$ m) for 50 Hz $\leq f \leq$ 800 Hz produced using a simple point source and the conventional and proposed approaches are plotted in Fig. 5. In Figs. 3 to 5, $20\log_{10}|S_{0+1}(x = 0, y = 0, z = 0.1)|$ was set at 0 dB for all frequencies.

These results indicate that both the conventional and proposed methods can generate 3D localized sound zones near the loudspeakers. Specifically, the proposed method's listening area ranges up to approximately $r_Q = 0.2$ m and the proposed method can also realize steeper sound pressure attenuation than the conventional approach at all frequencies. This means that S_0 can be more accurately cancelled when using the proposed method as compared to the conventional approach because S_0 in the proposed method only includes the 0-th order spherical harmonic spectrum component and is easily synthesized. Additionally, the proposed method does not require monopole pairs that include the approximation errors of dipole components.

Consequently, the proposed method can be simply realized using a practical planar omni-directional array with half the number of loudspeakers of the conventional method using monopole pairs [28]. Additionally, the results indicate that the listening area provided by the proposed method is more practical than those of the conventional evanescent and residual approaches [19–21, 23, 24].

Future work will include a detailed analysis of the relationships among the number of circular arrays, their radii and the listening area shape, and experiments using practically implemented arrays.

5. CONCLUSIONS

This paper has proposed a 3D localized sound zone generation method using a planar omni-directional loudspeaker array. In this approach, the sound field produced by the central loudspeaker is cancelled using multiple circular arrays and a 3D localized sound zone can then be generated. The resulting sound fields were decomposed into their spherical harmonic spectra and the driving function of the array was derived. The results of computer simulations validated the effectiveness of the proposed approach compared with the conventional method using monopole pairs.

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